

Quadratic equations

Strictly speaking a Quadratic equation arises from a pair of simultaneous equations:

If the simultaneous equations $y = x + 1$ and $y = 3x - 3$ are solved, we could eliminate the y and get $x + 1 = 3x - 3$, giving $x = 2$ and then $y = 3$. $(2,3)$ is the intersection of the two lines $y = x + 1$ and $y = 3x - 3$.

$y = x^2 + 8x - 20$ is a Quadratic curve and $y = 0$ is the equation of the x axis.
The intersection of the curve and the line is found by eliminating y and getting $x^2 + 8x - 20 = 0$.
This is a Quadratic equation.

Solving by trial and improvement: eventually we would get the two answers $x = 2$, $x = -10$.

Solving by graph: This would involve plotting the graph of $y = x^2 + 8x - 20$ and reading the intercepts on the x -axis.

Solving by factorising: If two expressions a and b are written $a \times b = 0$. Then a is 0 or b is 0.

$x^2 + 8x - 20 = 0$. may be written like this using the tool of factorising.

$$(x + 10)(x - 2) = 0$$

Then: $x + 10 = 0$ $x - 2 = 0$ putting each bracket equal to zero

Solutions: $\underline{x = -10}$ $\underline{x = 2}$

Solving by completing the square: This relies on us knowing what happens when we square a bracket. $(x + a)^2 = x^2 + 2ax + a^2$ the middle term is **twice** the **product** of the two terms in the bracket.

So $x^2 + 8x - 20$ can be written

$(x \quad)^2$ to give the x^2

Half the middle term must now go in

$(x + 4)^2$ to give the $+8x$

This unfortunately gives an extra 4^2 , so compensate:

$(x + 4)^2 - 16$

Put the constant term in

$(x + 4)^2 - 16 - 20$

Tidy up

$(x + 4)^2 - 36$ giving a **square** and some **spare**.

To solve the equation $x^2 + 8x - 20 = 0$, put

$(x + 4)^2 - 36 = 0$

So

$(x + 4)^2 = 36$

Square root:

$x + 4 = +6$ and -6 written ± 36

$x + 4 = 6$ gives $\underline{x = 2}$, $x + 4 = -6$ gives $\underline{x = 10}$.

Solving using the formula: This has similar steps to completing the square.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Identify a , b and c from the equation $ax^2 + bx + c$

Solve $3x^2 + 5x - 8 = 0$

$a = 3$, $b = 5$ and $c = -8$ now substitute into the formula:

$$x = \frac{-5 \pm \sqrt{(5^2 - 4 \times 3 \times (-8))}}{6} = \frac{-5 \pm \sqrt{(25 + 96)}}{6} = \frac{-5 \pm \sqrt{121}}{6}$$

Split into two answers: $\frac{-5 + 11}{6}$ or $\frac{-5 - 11}{6}$ $\underline{x = 1}$, $\underline{x = -2.33}$.

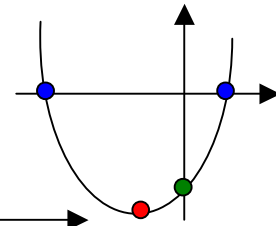


Uses for “completing the square”.

Completing the square can be used as a tool to find the lowest or highest point on a graph.

Example: $y = x^2 + 8x - 20$ is a Quadratic curve. When $x = 0$ $y = -20$
 When $y = 0$, $x = -10$ and 2 . (by factorising).

The coefficient of x^2 is positive so the **parabola** can hold water.



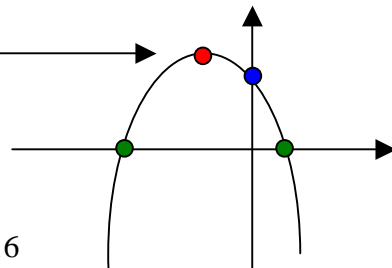
But what about the minimum point? →

After completing the square, $x^2 + 8x - 20$ may be written $(x + 4)^2 - 36$
 Since a **square** term can never be negative (even a minus squared is a plus), the lowest the square can be is zero. This happens when $x = -4$.
 This would make the lowest possible value of $(x + 4)^2 - 36$ to be -36 .
 The minimum point is $(-4, -36)$.

A curve which cannot hold water would be $y = -x^2 - x + 12$, since the coefficient of x^2 is negative.

When $x = 0$, y is **12**. When $y = 0$, $x = -4$ and $x = 3$

But what about the maximum point? →



Write $-x^2 - 2x + 15$ as $-(x^2 + 2x - 15)$
 $= -[(x+1)^2 - 1 - 15]$

$= -[(x+1)^2 - 16] = -(x+1)^2 + 16$

$= 16 - (x+1)^2$ the lowest the square can be is zero. When $x = -1$.

The highest possible value of $16 - (x+1)^2$ would be 16.

The maximum point is $(-1, 16)$.