Quadratic equations

Strictly speaking a Quadratic equation arises from a pair of simultaneous equations:

If the simultaneous equations $y = x + 1$ and $y = 3x - 3$ are solved, we could eliminate the y and get $x + 1 = 3x - 3$, giving $x = 2$ and then $y = 3$. (2,3) is the intersection of the two lines $y = x + 1$ and $y = 3x - 3$.

$y = x^2 + 8x - 20$ is a Quadratic curve and $y = 0$ is the equation of the x axis.
The intersection of the curve and the line is found by eliminating y and getting
$x^2 + 8x - 20 = 0$. This is a Quadratic equation.

Solving by trial and improvement: eventually we would get the two answers $x = 2$, $x = -10$.

Solving by graph: This would involve plotting the graph of $y = x^2 + 8x - 20$ and reading the intercepts on the x-axis.

Solving by factorising: If two expressions a and b are written a x b = 0. Then a is 0 or b is 0.
$x^2 + 8x - 20 = 0$. may be written like this using the tool of factorising.
$(x + 10)(x - 2) = 0$
Then: $x + 10 = 0$ | $x - 2 = 0$ putting each bracket equal to zero
Solutions: $\underline{x = -10}$ | $\underline{x = 2}$

Solving by completing the square: This relies on us knowing what happens when we square a bracket. $(x + a)^2 = x^2 + 2ax + a^2$ the middle term is **twice** the **product** of the two terms in the bracket.

So $x^2 + 8x - 20$ can be written $(x \quad)^2$ to give the x^2
Half the middle term must now go in $(x + 4)^2$ to give the $+8x$
This unfortunately gives an extra 4^2, so compensate: $(x + 4)^2 - 16$
Put the constant term in $(x + 4)^2 - 16 - 20$
Tidy up $(x + 4)^2 - 36$ giving a **square** and some **spare.**
To solve the equation $x^2 + 8x - 20 = 0$, put $(x + 4)^2 - 36 = 0$
So $(x + 4)^2 = 36$
Square root: $x + 4 = + 6$ and $- 6$ written ± 36
$x + 4 = 6$ gives $\underline{x = 2}$, $x + 4 = -6$ gives $\underline{x = 10.}$

Solving using the formula: This has similar steps to completing the square.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Identify a, b and c from the equation $ax^2 + bx + c$ Solve $3x^2 + 5x - 8 = 0$
$a = 3$, $b = 5$ and $c = -8$ now substitute into the formula:

$$x = \frac{-5 \pm \sqrt{(5^2 - 4\times3\times(-8))}}{6} = \frac{-5 \pm \sqrt{(25 + 96)}}{6} = \frac{-5 \pm \sqrt{(121)}}{6}$$

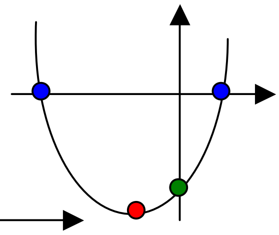
Split into two answers: $= \dfrac{-5 + 11}{6}$ or $\dfrac{-5 - 11}{6}$ $\underline{x = 1}$, $\underline{x = -2.33}$.

Uses for "completing the square".

Completing the square can be used as a tool to find the lowest or highest point on a graph.

Example: $y = x^2 + 8x - 20$ is a Quadratic curve. When $x = 0$ $y =$ **- 20**
When $y = 0$, $x =$ **-10 and 2**. (by factorising).

The coefficient of x^2 is positive so the **parabola** can hold water.



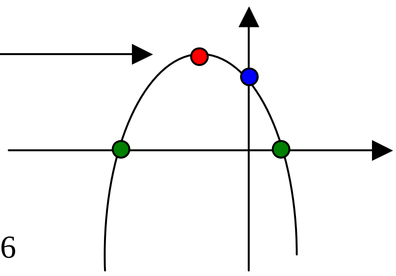
But what about the minimum point?

After completing the square, $x^2 + 8x - 20$ may be written $\mathbf{(x + 4)^2} - 36$
Since a **square** term can never be negative (even a minus squared is a plus), the lowest the square can be is zero. This happens when $\underline{x = -4.}$
This would make the lowest possible value of $(x + 4)^2 - 36$ to be $\underline{-36.}$
The minimum point is (-4, - 36).

A curve which cannot hold water would be $y = - x^2 -x +12$, since the coefficient of x^2 is negative.

When x = 0, y is **12.** When y = 0, x = **-4** and x = **3**

But what about the maximum point?



Write $- x^2 -2x +15$ as $-(x^2 +2x -15)$

$= \quad - \left[(x+1)^2 - 1 - 15) \right]$

$= \quad - \left[(x+1)^2 - 16 \right] \quad = \quad - (x+1)^2 + 16$

$= \quad 16 - (x+1)^2$ the lowest the square can be is zero. When x = -1.

The highest possible value of $16 - (x+1)^2$ would be 16.

The maximum point is (-1, 16).