HYPOTHESIS TESTING

At the lunchtime gambling club in the sixth form common room, it has been noticed that a particular student has been winning more often than expected, claiming that he has a lucky coin. The Statistician among the group decides to investigate this coin. The rest of the group are out for blood but the Statistician convinces them that a proper statistical test needs to be done and they will have to wait for the result before any decision is to be made on banning the alleged cheat.
The 'lucky' coin is to be taken, tossed 10 times and the results are to be recorded as the number of heads that occur.

The group are getting excited; they can't wait to do some serious damage to the offender. One of them cries out, "If we get 10 heads out of 10 then it's curtains for him". Another one cries out, "Why do we have to buy him curtains? He needs beating up". "Shut up stupid", another one cries, "or we will beat **you** up".
"Anyway," the Statistician says, "We have to use proper statistical techniques not gut feelings".
So let us toss the coin and see what occurs.

First we need a **null hypothesis** and this can be that the probability of a head is $\frac{1}{2}$ i. e. H_0: Pr(head) = $\frac{1}{2}$

This is tested against the **alternate hypothesis** that the coin is biased in favour of heads: H_1: Pr(head) > $\frac{1}{2}$

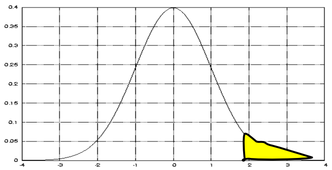
We are looking to **reject** the null hypothesis if we want to find the offender guilty of using a coin, which is biased in favour of heads.

Based on the null hypothesis, a picture of all the possible outcomes with associated probabilities can be built up. A probability distribution.

Number of heads	0	1	8	9	10
probability	$\binom{10}{0}\left(\frac{1}{2}\right)^{0}\left(\frac{1}{2}\right)^{10}$	$\binom{10}{1}\left(\frac{1}{2}\right)^{1}\left(\frac{1}{2}\right)^{9}$	$\binom{10}{8}\left(\frac{1}{2}\right)^{8}\left(\frac{1}{2}\right)^{2}$	$\binom{10}{9}\left(\frac{1}{2}\right)^{9}\left(\frac{1}{2}\right)^{1}$	$\binom{10}{10}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^{0}$
Percentage probability	0.001	0.01	0.044	0.01	0.001

The probability of actually getting 10 heads, if the coin is fair, is 0.001. That is one in a thousand. So the occurrence of 10 heads is **highly unlikely** this occurrence would give us reasonable justification to go ahead and find the alleged offender guilty.
The probability of 9 heads is 0.01 and this too is considered statistically significant. **By significant, we mean a probability of less than 5%.**

The probability of getting 8 heads is **less than 5%** (0.04), this too seems to give us reasonable cause to reject the null hypothesis and start dishing out the punishment.
In fact, we do not take this probability in isolation but look at the probabilities in the entire tail. That is, we look at **cumulative probabilities.**



Working from the right of the table: Pr(10 heads) = 0.001, Pr(9 or 10 heads) = 0.011 and Pr(8, 9, or 10 heads) = 0.055
This figure exceeds the 5% margin that we consider significant, so we only conclude that a result of 9 or 10 heads leads to a rejection of the null hypothesis.
Any other result means we cannot reject H_0 and have no grounds to carry out any punishment.
Of course, if we were that desperate for blood, we could have decided to use a 10% significance level. A result of 8 heads would then be included in the set of significant results. Perhaps even 7 heads would be considered significant?

This is how to tackle a problem:

Over a long period of time it has been found that the proportion of "Doner" Kebabs sold from Jimala's Kebab Van in a day is 1 in 4. I think that since the new pub has opened down the road the customer's taste for Doner Kebabs has increased. Donner Kebabs go down well after a pint.
To test this theory I made a record of 20 kebabs sold on one particular evening and found that 10 were "Doner" Kebabs. Carry out a hypothesis test at the 5% level of significance and state an assumption you need to make when carrying out the test.

H_0: Pr(donner) = $\frac{1}{4}$ the proportion of doner kebabs sold is 1 in 4. With a Binomial distribution, the mean or expected number is ¼ x 20 = 5.

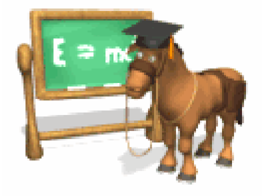
H_1: Pr(donner) > $\frac{1}{4}$ this is a one-tailed test. We are looking for a high proportion of doner kebabs sold if we are to reject H_0.

The result of the test sample was **10** doner kebabs. We have to look at the probability of getting 10 or more successes with p = ¼ and n = 20 from the Binomial distribution. From Binomial tables $Pr(x \geq 10) = 1 - Pr(x \leq 9) = 1 - 0.9861 = 0.0139$
and this is less than 5% so it is considered a highly unlikely event.

Since $0.0139 < 0.05$, we reject the null hypothesis and conclude that there is enough evidence to suggest that the proportion of doner kebabs sold has increased.

If we look at the tables again we see that even a result of **9** would lead us to reject the null hypothesis since $\Pr(x \geq 9) = 0.0409$
The assumption we have made is that the results were recorded on a typical day and not on something like "World Doner Kebab day".

HYPOTHESIS TESTING USING A POISSON DISTRIBUTION



The number of donkeys taken in by the local donkey sanctuary in a year in Prussia has been 2 on average. The sanctuary believes that the incidences of cruelty to donkeys have increased and records show that a total of 11 donkeys were taken in over the last three years. A hypothesis test will be performed to determine if there has been an increase in donkey cruelty.

Events, which occur independently in time, follow a Poisson distribution and this is no exception. The mean is 2 per year or 6 in a period of 3 years.

The **null hypothesis** can be that the mean is equal to 6. $H_0: \lambda = 6$
The **alternate hypothesis** is that the mean is greater than 6. $H_1: \lambda > 6$ (a one-tailed test)

We are looking to **reject** the null hypothesis if we want to conclude that there has been an increase in donkey cruelty.

From the Poisson probability distribution with $\lambda = 6$, the probability of 11 or more successes needs to be found. $\lambda = 6$ can't be found in tables.

$$\Pr(r \geq 11) = 1 - \Pr(r \leq 10) = 1 - \sum_{r=1}^{r=10} \frac{e^{-\lambda}\lambda^{r}}{r!} = 0.0426$$

this is less than 5% so it is considered a highly unlikely event

Since $0.0426 < 0.05$, we reject the null hypothesis and conclude that there is enough evidence to suggest that donkey cruelty has increased.

After a campaign by the sanctuary, the tree months following the campaign saw only 2 donkeys brought in. Does this mean that the campaign was successful in the decrease of cruelty to donkeys? $H_1: \lambda < 6$ (a one-tailed test again)

This time we look at the left hand tail of the distribution and observe that $\Pr(r \leq 2) = 0.0620$. Since $0.0620 > 0.05$, the result is not considered "highly unlikely" and we say we have no grounds to reject the null hypothesis.