THE BINOMIAL E X P A N S I O N

You should already know how to square a bracket $(a + b)$

$(a + b)^2 = (a + b)(a + b) = a^2 + ab + ba + b^2 = a^2 + 2ab + b^2$

Cubing a bracket: $(a + b)^3 = (a + b)(a + b)^2 = a^3 + 3a^2b + 3a^2b^2 + b^3$

Continuing: $(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

Continuing: $(a + b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

Continuing: $(a + b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$

Looking at the pattern of coefficients for these expansions and putting $(a + b)^0$ at the top.

This pattern of numbers is known as **Pascal's triangle,**

devised by the French Mathematician Blaise Pascal.

```
                1
              1   1
            1   2   1
          1   3   3   1
        1   4   6   4   1
      1   5   10  10  5   1
    1   6   15  20  15  6   1
```

The next row can be obtained by starting with 1, adding the two numbers above and ending with 1

1 **7** 21 35 35 21 7 1

Notice the powers of **a** get smaller and the powers of **b** are growing with consecutive terms

So $(a + b)^7$ will be $a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7$

GENERAL TERMS OF THE EXPANSION

How about finding the **fifth** term of $(a + b)^{11}$?

Counting down: 11, 10, 9, 8, 7

Since 7 + **4** = 11,

The first term must be a^{11}

the fourth term contains a^7

the fourth term contains b^4

Finally, the coefficient may be found by continuing Pascal's triangle up to the line 1 11 …….

…… but a quicker way is to write ${}^{11}C_4$ or $\binom{11}{4}$ then using "4 numbers down" $\frac{11\text{x}10\text{x}9\text{x}8}{1\text{x}2\text{x}3\text{x}4}$ divided by "4 numbers up"

this cancels down to 330 and so the **fourth** term is $330\,a^7b^4$

$\binom{11}{4}$ may be also found using factorial notation (!) with your calculator $\boxed{\frac{11!}{4!7!}} = 330$

note: $\binom{11}{7}$ is $\boxed{\frac{11!}{7!4!}}$ which is also 330 so to find the term containing $\mathbf{a^4}$, the eighth term,

write a^4b^7 then introduce the coefficient $\binom{11}{7}$ or $\binom{11}{7}$ at the head of the term……$330\,a^4b^7$

The coefficients on any row of Pascal's triangle have symmetry. E.g. The 6th row: 1 5 10 10 5 1

The 7th row: 1 6 15 20 15 6 1

note:

The expansion of $(x + \frac{1}{x})^6$ has middle term $20x^3(\frac{1}{x})^3 = 20$

And this term is independent of x.