

THE BINOMIAL EXPANSION

You should already know how to square a bracket

$$(a+b)^2 = (a+b)(a+b) = a^2 + ab + ba + b^2 = a^2 + 2ab + b^2$$

Cubing a bracket:

$$(a+b)^3 = (a+b)(a+b)^2 = a^3 + 3a^2b + 3a^2b^2 + b^3$$

Continuing:

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

Continuing:

$$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

Continuing:

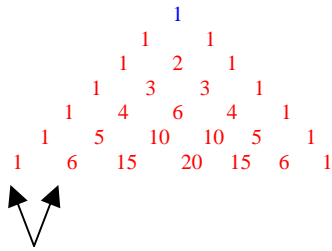
$$(a+b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$$

Looking at the pattern of coefficients for these expansions and putting $(a+b)^0$ at the top.

This pattern of numbers is known as **Pascal's triangle**,



devised by the French Mathematician **Blaise Pascal**.



The next row can be obtained by

starting with 1, adding the two numbers above and ending with 1

Notice the powers of **a** get smaller and the powers of **b** are growing with consecutive terms

$$1 \quad 7 \quad 21 \quad 35 \quad 35 \quad 21 \quad 7 \quad 1$$

So $(a+b)^7$ will be

$$a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7$$

GENERAL TERMS OF THE EXPANSION

How about finding the **fifth** term of $(a+b)^{11}$?

Counting down: 11, 10, 9, 8, 7

Since $7 + 4 = 11$,

The first term must be

$$a^{11}$$

the fourth term contains

$$a^7$$

the fourth term contains

$$b^4$$

Finally, the coefficient may be found by continuing Pascal's triangle up to the line **1 11**

..... but a quicker way is to write ${}^{11}C_4$ or $\binom{11}{4}$ then using "4 numbers down" $\frac{11 \times 10 \times 9 \times 8}{1 \times 2 \times 3 \times 4}$ divided by "4 numbers up"

this cancels down to **330** and so the **fourth** term is

$$330 a^7 b^4$$

$\binom{11}{4}$ may be also found using factorial notation (!) with your calculator $\frac{11!}{4!7!} = 330$

note: $\binom{11}{7}$ is $\frac{11!}{7!4!}$ which is also 330 so to find the term containing a^4 , the eighth term,

write $a^4 b^7$ then introduce the coefficient $\binom{11}{7}$ or $\binom{11}{4}$ at the head of the term..... $330 a^4 b^7$

The coefficients on any row of Pascal's triangle have symmetry. E.g. The 6th row: 1 5 10 10 5 1
The 7th row: 1 6 15 20 15 6 1

note:

The expansion of $(x + \frac{1}{x})^6$ has middle term $20x^3(\frac{1}{x})^3 = 20$

And this term is independent of x .