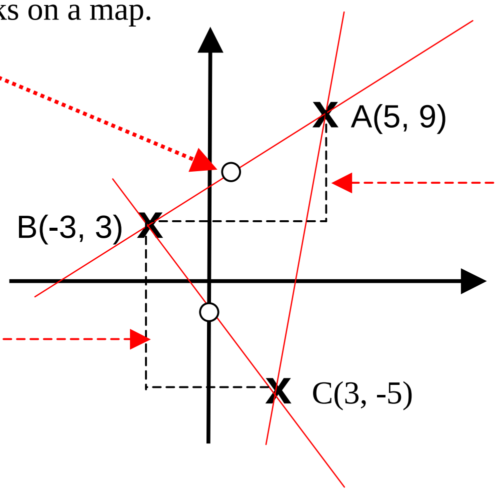
(Co, ordinate) Geometry

Consider three points A(5, 9), B(-3, 3) and C(3, -5).
These could be three landmarks on a map.

The mid-point of AB is found using the average or **mean** of the co-ordinate values:

	5	9	
	-3	3	+
Totals	2	12	divide by 2
Mean	1	6	
Mid-point	(1, 6)		

The distance between any two points is found using Pythagoras' Theorem.
$BC^2 = 8^2 + 6^2$
A sketch graph shows the y-step is 8 and the x-step is 6.
BC = 10 Where is the mid-point?



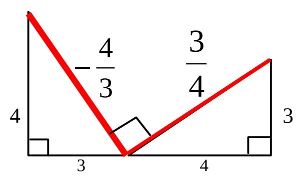
The gradient of a line passing through two given points is defined as:

$$\text{Gradient} = \frac{y \cdot step}{x \cdot step} = \frac{6}{8}$$

The gradient of the line through A and B is $\frac{3}{4}$

The line through B and C has a gradient of $-\frac{8}{6} = -\frac{4}{3}$

Two gradients are perpendicular if their product is –1. Or you can turn the fraction upside down and change the sign.



The following are pairs of perpendicular gradients.

$\frac{3}{4}$	$\frac{4}{7}$	$-\frac{2}{3}$	2 $\left(\frac{2}{1}\right)$	$-\frac{1}{3}$
$-\frac{4}{3}$	$-\frac{7}{4}$	$\frac{3}{2}$	$-\frac{1}{2}$	3

The equations of lines.

The line through B and C has gradient $-\frac{4}{3}$ and since the equation of any line is **y = mx + c,**

The line must have equation of the form: **y =** $-\frac{4}{3}$ **x + c.** To complete the equation we have to find **c.**

We know the line passes through **C(3, –5)** therefore substitute x = 3 and y = –5 into the equation:

$-5 = -\frac{4}{3}3 + c$ **This gives c = –1.** **The line through B and C has equation** $y = -\frac{4}{3}x - 1$

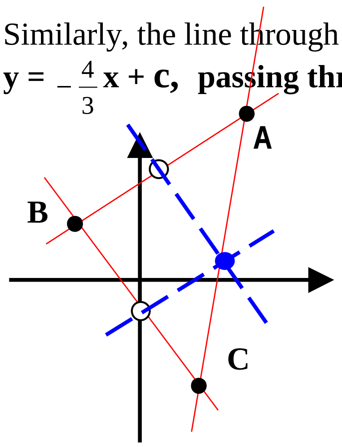
Any line **perpendicular** to the line through B and C will have gradient equal to $\frac{3}{4}$

The equation of any such line will be of the form ……….. $y = \frac{3}{4}x + c.$

To find the equation of the perpendicular line through the mid-point of BC, substitute the co-ordinates of the mid-point. **This point is (0, –1).** $-1 = \frac{3}{4}0 + c$ $c = -1$

The equation of the perpendicular line is $y = \frac{3}{4}x - 1$

Similarly, the line through the mid-point of AB and perpendicular to AB has equation:
$y = -\frac{4}{3}x + c$, **passing through (1, 6).** $6 = -\frac{4}{3}1 + c$ $y = -\frac{4}{3}x + \frac{22}{3}$



The point of intersection of these two perpendicular lines will be the centre of the circle, which can be drawn to go through the vertices of the triangle ABC.
(To find the point using compasses we would have to draw the perpendicular bisectors of AB and BC.
Solving the equations simultaneously will also give that point.

The two perpendicular lines have equations $y = \frac{3}{4}x - 1$ **and** $y = -\frac{4}{3}x + \frac{22}{3}$ **respectively.**

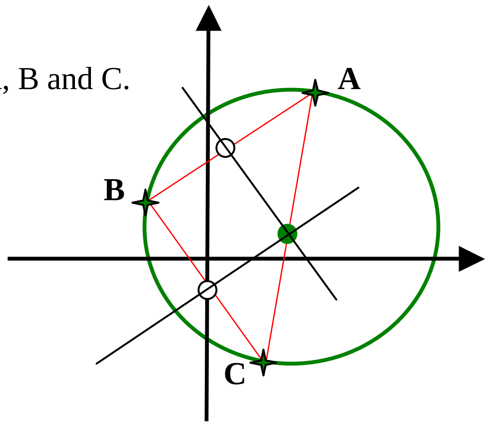
Multiplying the first by 4 and the second by 3 would clear the fractions and rearranging means we could write the equations in the form **ax + by = c.**

$4y - 3x = -4$ (i) $\xrightarrow{\times 4}$ $16y - 12x = -16$ $\xrightarrow{\text{add}}$ $25y = 50$ $\quad y = 2,\ x = 4$

$3y + 4x = 22$ (ii) $\xrightarrow{\times 3}$ $9y + 12x = 66$

With the compass point at (4, 2)
a circle can be drawn to go through A, B and C.

This is known as the circum-circle.



POSITIONING A LINE ON THE AXES

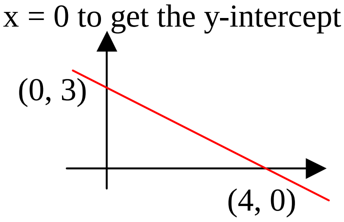
To correctly position the line $y = 2x + 3$: set up a table with your own choice of x values and use the formula $2x + 3$ to obtain the y values.

x	**1**	**3**	**5**
y	**5**	**9**	**13**

plot the points (1,5) (3,9) (5,13) and draw the line through these points. There will be lots of other points on this line eg. (7,17).

Or we could observe that the gradient is 2 and the intercept on the y-axis is (0, 3).

To correctly position a line of the form ax + by = c on the axes, put x = 0 to get the y-intercept and then put y = 0 to get the x-intercept.



3x + 4y = 12 cuts the co-ordinate axes at (0, 3) and (4, 0).

To find the gradient and intercept if the line is given in the form ax + by = c rearrange the equation into the form $y = -\frac{a}{b}x + \frac{c}{b}$.

<u>Rieley's method.</u> To find the equation of a line with gradient $\frac{a}{b}$, passing through (h, k):

Using y = mx + c: $y = \frac{a}{b}x + c$

Substitute x = h, y = k. $k = \frac{a}{b}h + c$, $c = k - \frac{a}{b}h$ $\qquad c = \frac{bk - ah}{b}$

The equation is $y = \frac{a}{b}x + \frac{bk - ah}{b}$

Multiplying throughout by b and rearranging: $ax - by = ah - bk$

I notice that **a** goes with **x** , **b** goes with **y** and the sign is changed. The right hand side is a copy of the left with h and k substituted into x and y respectively.

Gradient $\frac{3}{4}$, passing through (5, 2) $\quad 3x - 4y = 3(5) - 4(2)$ $\quad$ **<u>3x – 4y = 7</u>**

Gradient $-\frac{2}{3}$, passing through (-4, 6) $\quad 2x + 3y = 2(-4) + 3(6)$ $\quad$ **<u>2x + 3y = 10</u>**