PROBABILITY

The probability of an event happening is written P(event)

The answer (usually a fraction) is worked out using $\dfrac{\textbf{Number of favourable outcomes}}{\textbf{Total number of outcomes}}$

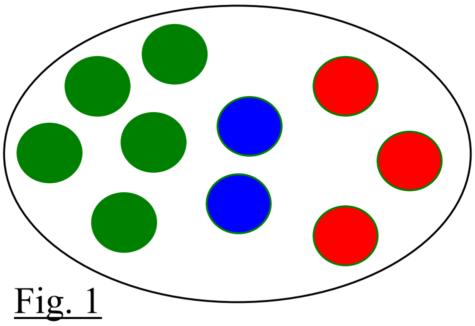


Fig. 1

The events "green disc" and "red disc" are said to be **mutually exclusive** because both events cannot occur at the same time if we pick **one** disc.
The probability of picking a red **or** green is

$$\frac{3}{10} + \frac{5}{10} = \frac{8}{10}$$

"or" events mean we **add** probabilities.
The symbol used is **"union"** U and we write:

P(A U B) = P(A) + P(B)

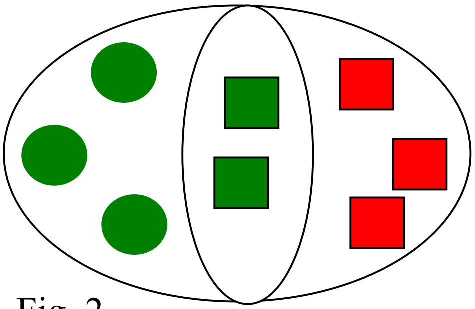


Fig. 2

The events "green disc" and "square disc" are **not mutually exclusive** because of the chance of a **square green** disc.
The probability of picking a green square disc is the probability of picking a disc that is both green **and** square.

The symbol used is **"intersection"** ∩ and we write:

$$P(A \cap B) = \frac{1}{4}$$

$$\textbf{P(green and square)} = \frac{2}{8} = \frac{1}{4}.$$

This is also P(green) + P(square) - P(green Square).
Since the green squares have been included twice.
In this case we write: **P(A U B) = P(A) + P(B) - P(A ∩ B)**

$$= \frac{5}{8} + \frac{5}{8} - \frac{2}{8} = 1$$ since we are certain to get a square or a green.

If A and B are mutually exclusive as in Fig. 1, then **P(A ∩ B) = 0**

CONDITIONAL PROBABILITY

If two discs are taken without replacement from **figure 1**,then the probability of two green discs is the probability of a green disc followed by another green disc.
P(green and green) can be written **P(green first ∩green second) using the intersection symbol.**
This is worked out using **P(green first) x P(green second, with the condition that green was first)**

$$= \frac{5}{10} \times \frac{4}{9}$$

Symbolically this is **P(A ∩ B) = P(A)xP(B|A)** and for any two events this is also
= P(B)xP(A|B)

If the second event is not conditional on the first, the two events are <u>independent</u> and
P(B|A) = P(B) the occurrence of A does not effect the probability of B.
P(A ∩ B) = P(A)xP(B|A) becomes

P(A ∩ B) = P(A)xP(B)

This is known as the multiplication law for independent events.

Consider a fairground stall with three guns A, B and C. You are equally likely to pick any gun.
However, the probabilities of hitting the target are different for each gun.

$$P(hit|A) = \frac{1}{4}, \quad P(hit|B) = \frac{1}{2}, \quad P(hit|C) = \frac{1}{3},$$

The probability you pick gun A and hit the target: $P(A \cap hit) = P(A) x P(hit|A) = \frac{1}{3} x \frac{1}{4}$

The overall probability of a hit is (A and hit) or (B and hit) or (c and hit).

$$P(hit) = P(A)xP(hit|A) + P(B)xP(hit|B) + P(C)xP(hit|C) = \frac{1}{3} x \frac{1}{4} + \frac{1}{3} x \frac{1}{2} + \frac{1}{3} x \frac{1}{3} = \frac{13}{36}$$

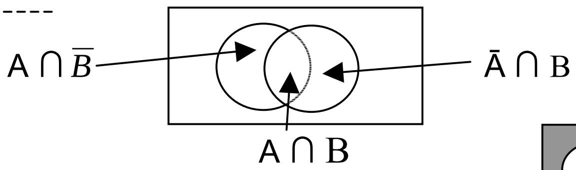
What if we have chosen a gun and just hit the target? What is the probability that it was gun B? This seems like we have to go *back in time*, but the answer may be found using conditional probability.

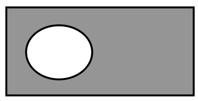
P(A ∩ B) = P(A)xP(B|A) may be rearranged to give:

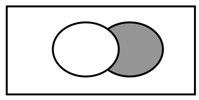
$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(B) \times P(A|B)}{P(A)}$$

SO: $P(B|hit) = \frac{P(B \cap hit)}{P(hit)} = \frac{P(B) \times P(hit|B)}{P(hit)} = \frac{\frac{1}{3} \times \frac{1}{2}}{\frac{13}{36}} = \frac{6}{13}$ *real Doctor Who stuff..*

Dashed events!!







P(Ā) is the probability of A not occurring i.e. **1 – P(A)**

P(Ā∩B) is **P(B) - P(A∩B)**

P(Ā|B) + P(A|B) = 1 since we can either **have A** or **not have A**, given B.

The laws usually used in exam questions are:

P(A U B) = P(A) + P(B) for mutually exclusive events.
P(A U B) = P(A) + P(B) - P(A ∩ B) for non-mutually exclusive events.
IF **P(A ∩ B) = 0 then A and B are mutually exclusive.**

P(A ∩ B) = P(A)xP(B|A) for any two events which are not independent.
P(A ∩ B) = P(A)xP(B) for any two events which are **independent**. Where **P(B|A) = P(B)**

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

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