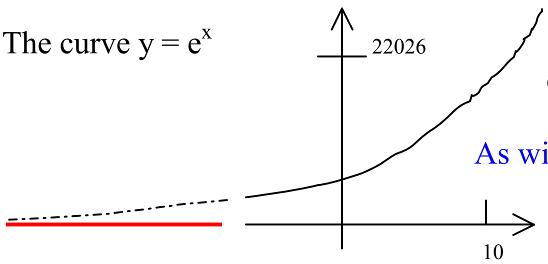
EXPONENTIALS & LOGARITHMS

IN ADDITION TO THE SKETCH CURVES YOU WILL HAVE ALREADY MEMORISED, You will need to memorise two more for this section:

The curve $y = e^x$



$e^1 = e$ which has the value 2.718……….

$e^{10} \approx 22026$ using a calculator. **Note the distorted scale!**

As with any index, $e^0 = 1$ (the curve passes through (0,1)

Since $e^{-\infty} = \dfrac{1}{e^{\infty}} = 0.$

The curve touches the x-axis at $-\infty$
The curve is never negative.

The x-axis is an **asymptote** to the curve.

This topic is linked to <u>functions</u> so $e^{(x-a)}$ is the e^x graph translated by **a** units to the right. **a** ⟶

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LOGARITHMS

A **logarithm** or log is simply a power or **index**.

$1 = 10^0$
$10 = 10^1$
$100 = 10^2$
$1000 = 10^3$

The numbers have all been expressed as powers of 10.
10 is the base

log to the base 10 of the number 1 is zero $\log_{10} 1 = 0$
log to the base 10 of the number 10 is one $\log_{10} 10 = 1$
log to the base 10 of the number 100 is two $\log_{10} 100 = 2$
log to the base 10 of the number 1000 is three $\log_{10} 1000 = 3$

A number like 600 is $10^{2.\text{ something}}$ log to the base 10 of the number 600 is 2.something $\log_{10} 600 = 2\,.\sim\sim$

So if $91 = 3^4$ then $\log_3 91 = 4$ and

if $a = b^c$ then $\log_b a = c$

Fortunately, we normally only use one of two bases. Base **e** or base **10** and we write:
$\log_e x$ or lnx to stand for **n**atural **l**og. We may also use $\log_{10} x$ or simply logx when base 10 is used.

<u>The rules for working with logs</u>

<u>Adding logs</u> $\log_b a + \log_b c$

let $\log_b a = p$ and $\log_b c = q$
(change to indices) then $a = b^p$ and $c = b^q$
(combine using the rules of indices) $ac = b^p\, b^q = b^{p+q}$
(revert to logs) $\mathbf{\log_b ac = p + q = \log_b a + \log_b c}$

<u>Subtracting logs</u> $\log_b a - \log_b c$

let $\log_b a = p$ and $\log_b c = q$
(change to indices) then $a = b^p$ and $c = b^q$
(combine using the rules of indices) $a/c = b^p/b^q = b^{p-q}$
(revert to logs) $\mathbf{\log_b (a/c) = p - q = \log_b a - \log_b c}$

Addition

Subtraction using the same steps above with a minus

Powers $\log_b a^3 = \log_b aaa = \log_b a + \log_b a + \log_b a$

Negatives $\log_b a - \log_b c = -(\log_b c - \log_b a)$

Any base $\log_3 7$ cannot be found directly but using ………

$$\log_b a + \log_b c = \log_b ac$$

$$\log_b a - \log_b c = \log_b \frac{a}{c}$$

$$\log_b a^n = n\log_b a$$

$$\log_b \frac{a}{c} = -\log_b \frac{c}{a}$$

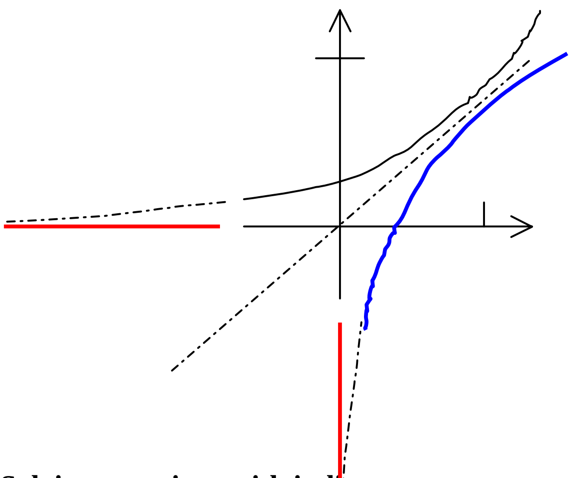
$$\log_b a = \frac{\log_c a}{\log_c b}$$

And changing to base e or 10 $\log_3 7 = \dfrac{\log_{10} 7}{\log_{10} 3} = 1.77$ this means that $3^{1.77} = 7$.

Now consider the function $y = e^x$ again: $\mathbf{f(x) = e^x}$, **find $\mathbf{f^{-1}(x)}$**

let	$y = e^x$	
take logs	$\ln y = \ln e^x$	$\ln y = x\ln e$
write as indices	$x = \ln y$	since $\ln e = 1$
interchange	$y = \ln x$	
so	$f^{-1}(x) = \ln x$	

this is the **inverse function** and is the original function reflected in the line **y = x.**



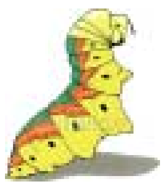
This is the graph of $y = \ln x$ memorise it!
It does not exist for negative values of x.
The y-axis is an asymptote to the curve.
The asymptote will change if the function is transformed.
For example $y = \ln(x - 2)$ will be the graph of $\ln x$ translated 2 units to the right. The new asymptote will be the line $x = 2$.

<u>Solving equations with indices</u>

To solve $3^x = 7$.

Take logs of both sides $\log 3^x = \log 7$

Move the index down and cross multiply $x \log 3 = \log 7$, $x = \dfrac{\log 7}{\log 3} = 1.77$



The probability of finding a particular type of bug on an exotic leaf is 0.1. If I have to travel to the other side of the world to collect the leaves, how many should I collect so that I can be 95% certain that I will have at least one bug when I get home?

Pr (bug) = 0.1 If I collect **n** leaves, then the number of bugs I could get may be modelled by the Binomial distribution with p = 0.1 and q = 0.9.

Pr(at least 1) = 1 - Pr(none) = $1 - (0.9)^n$ which must be 0.95 for 95% certainty

$$1 - (0.9)^n = 0.95$$

$$0.05 = (0.9)^n$$

take logs $\log 0.05 = n \log 0.9$, $\dfrac{\log 0.05}{\log 0.9} = n$, $n = 28.4$

Bring back 29 leaves just to be sure!

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