Functions

A function is like a wedding. There are tables, chairs, food, wine etc.

A function of x tells the story of the output for any input value of x.   Written $f : x \mapsto x^2 + 2x - 8$

Or $f(x) = x^2 + 2x - 8$

Put x = 3 and we get $f(3) = 3^2 + 2x - 8 = 7$.   (3, 7) is a point on the graph of $y = x^2 + 2x - 8$.

COMPOSITE FUNCTIONS

Composites occur when mixing, for example in making mortar from sand and cement. Composite functions occur when mixing functions.

If $f(x) = 2x – 1$ and another function $g(x) = x + 3$ are given:

The composite functions fg(x) and gf(x) may be formed.

f(**x**) = 2**x** – 1, f(**5**) = 2(**5**) – 1, so   fg may be taken as f with the entire g-function substituted in:
fg(x) =   2(**x + 3**) – 1, which forms a new function fg(x) = 2x + 5.

We can form many new functions and find values for particular values of x.

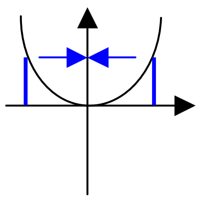
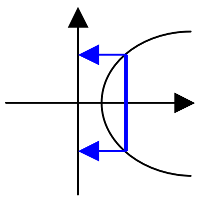
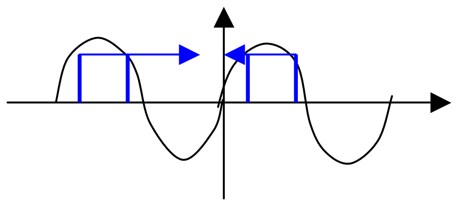
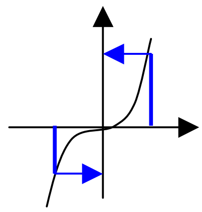
gf(x) = (**2x +1**) + 3 , substituting the f-function into the g-function: gf(x) = 2x + 4.

Note that   fg(x)   ≠   gf(x).   WE can even find fgf(x): here it is better to work backwards

by first finding gf and substituting into the f-function. f **g f(x)** = 2(**2x + 4**) – 1 = 4x + 7.

INVERSE FUNCTIONS

Inverse functions can only exist for functions which are **one-to-one functions.** Let us call these **Pure Functions.**

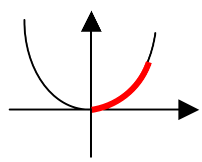





This is a many-to-one function. Two values of x give the same (one) value of y.	This is a one-to-many function. One value of x gives two different values of y.	This is a many-to-one function. Several values of x give the same (one) value of y.	**This is a one-to-one function. Every <u>one</u> value of x gives only <u>one</u> value of y and vice versa. This function has an inverse.**

DOMAIN AND RANGE

These functions do not have an inverse because they are not one-to-one.
They may however, have an inverse if a certain part of the curve is considered and defined as the **DOMAIN**.

DOMAINS: $0 \le x \le 3$ $x \in \mathbb{R}$ (where R is the set of real numbers).
The **DOMAIN** is the set of x values where the curve section exists. ($\in$)
The **RANGE** is the corresponding set of y values.
For $f(x) = x^2$ with domain $0 \le x \le 3$ the range is written $0 \le y \le 9$



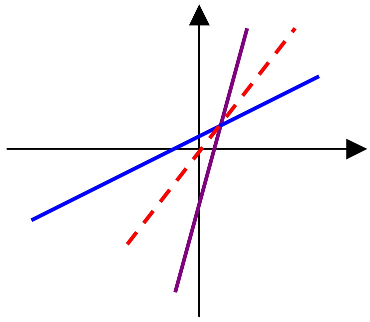
Domain $0 \le x \le 3$

FINDING THE INVERSE AND SKETCHING THE GRAPHS

f(x) = 2x – 1 $x \in \mathbb{R}$ To find the inverse let $y = 2x - 1$

Rearrange to make x the subject $x = \dfrac{y+1}{2}$

Interchange x and y and write this as a function of x **$f^{-1}(x)$ =** $\dfrac{x+1}{2}$



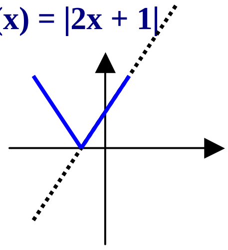
Notes:
The inverse of f(x) is written **$f^{-1}(x)$.**
$f^{-1}(x)$ will always be the reflection of f(x) in the x-axis.
The intersection of f(x) and $f^{-1}(x)$ will be on the line y = x.
When solving f(x) = $f^{-1}(x)$. Simply solve f(x) = x.

THE MODULUS FUNCTION

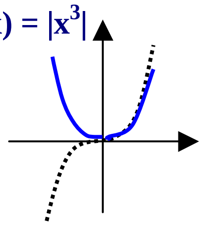
| f(x) | THIS IS MOD f(X) AND TURNS ALL NEGATIVE Y VALUES POSITIVE.

To sketch a modulus function, first sketch (dotted) the full function then over-sketch with all negative values being reflected in the x-axis.

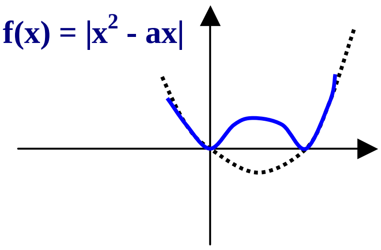
f(x) = |2x + 1|



f(x) = $|x^3|$



f(x) = $|x^2$ - ax|



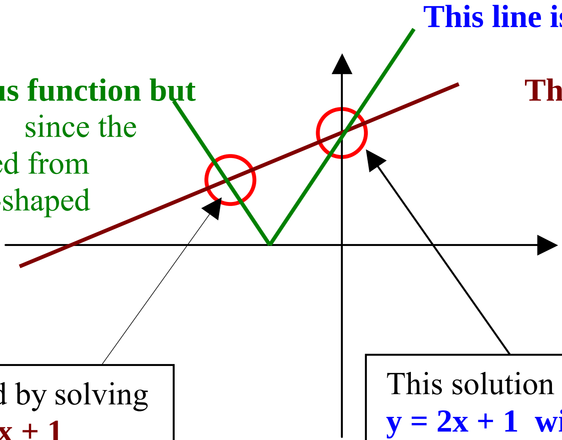
To solve equations containing modulus functions sketch both functions first.

Solve **|2x + 1| = ½x + 1**

This line is y = 2x + 1

This line is part of the modulus function but take it as the Line y = -(2x + 1); since the line y = 2x + 1 has been reflected from below the x axis to create the v-shaped modulus function.

This is the line y = ½x + 1

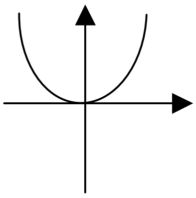


This solution can be found by solving
y = -(2x + 1) with y = ½x + 1
-(2x + 1) = ½x + 1
-2x – 1 = ½x + 1
-2 = 2½x
x = - $\dfrac{4}{5}$

This solution can be found by solving
y = 2x + 1 with y = ½x + 1
2x + 1 = ½x + 1
1½x = 0
x = 0
Substituting x = 0 into either of the above equations gives y = 1.

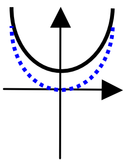
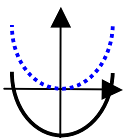
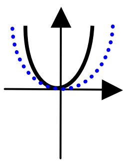
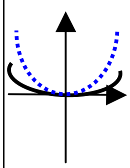
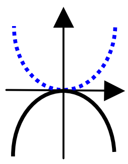
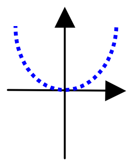
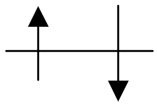
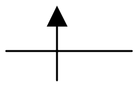
Transformations

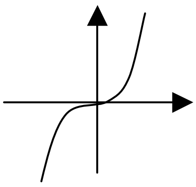
Functions can be **transformed** by changing the form of the original function.



This is the basic curve $y = x^2$. It is called a parabola and it can "hold water".

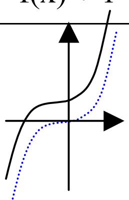
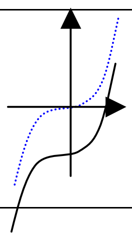
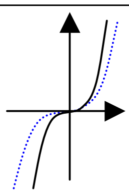
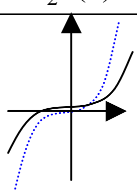
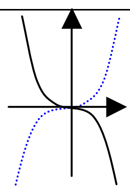
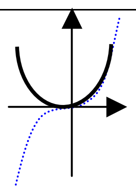
We write $f(x) = x^2$ This is the simplest quadratic and it can be transformed:

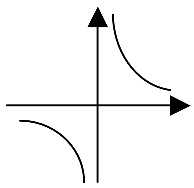
EXTERNAL TAMPERING	$f(x) + 1$	$f(x) - 2$	$2f(x)$	$\frac{1}{2}f(x)$	$-f(x)$	$\mid f(x) \mid$
EFFECT	THE +1 TAKES THE GRAPH 1 UNIT UP.	THE -2 TAKES THE GRAPH DOWN 2 UNITS.	MULTIPLYING BY 2 TAKES EVERY Y VALUE TWICE AS FAR FROM THE X-AXIS.	THIS EFFECT SHRINKS EVERY VALUE NEARER TO THE X-AXIS BY A FACTOR OF $\frac{1}{2}$	THIS EFFECT REVERSES THE SIGN OF THE Y VALUES. NOW $Y = -x^2$ Reflection in the x-axis	THIS IS MOD F(X) AND TURNS NEGATIVE Y VALUES POSITIVE. Already positive here so no change
The original curve is dotted						
CODE	↑ 1	↓ -2	↕ X2	↓↑ X $\frac{1}{2}$		



This is the basic curve $y = x^3$.

These are the effects of the same transformations:

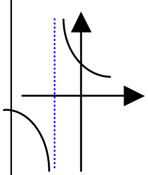
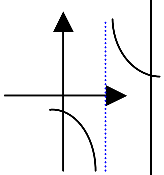
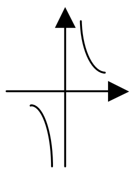
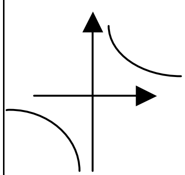
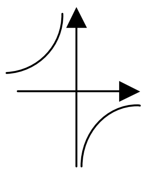
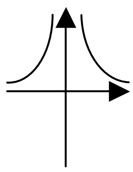
EXTERNAL TAMPERING	$f(x) + 1$	$f(x) - 2$	$2f(x)$	$\frac{1}{2}f(x)$	$-f(x)$	$\mid f(x) \mid$
						



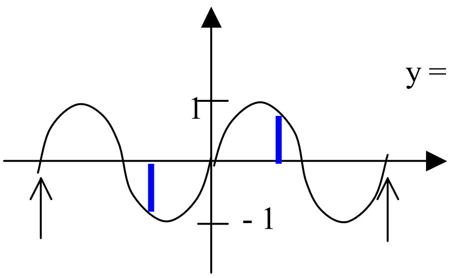
This is the basic curve $y = \frac{1}{x}$

The axes are **asymptotes** to the curve. The curve touches the axes at infinity.

We will **transform** this curve with some internal tampering.

(EXTERNAL)	$f(x) + 1$	$f(x) - 2$	$2f(x)$	$\frac{1}{2}f(x)$	$-f(x)$	$\lvert f(x) \rvert$
INTERNAL TAMPERING	$f(x + 1)$	$f(x - 2)$	$f(2x)$	$f(\frac{1}{2}x)$	$f(-x)$	$f(\lvert x \rvert)$
All transformations happen in a horizontal direction and are **opposite** to Expectation.						
CODE	← Back 1	→ Forward 2	→ ← Shrink 2	↔ Expand 2	Reflect in y - axis	Who knows?

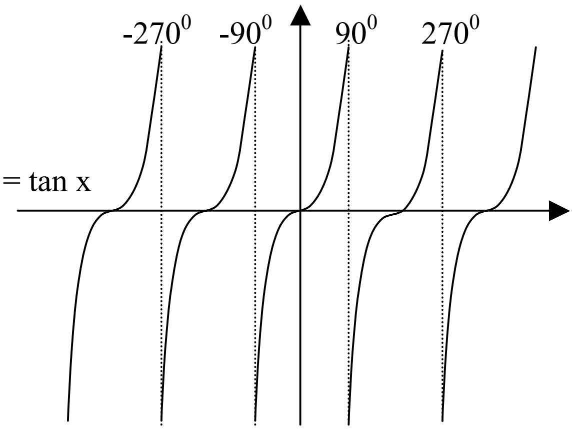
Just a few other curves which you may be asked to transform:



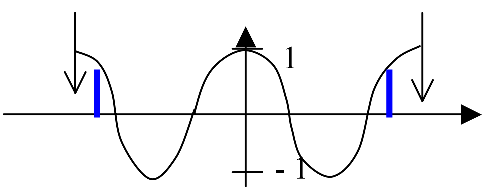
y = sin x

The sine function is an odd function
$\sin(-x) = -\sin(x)$

y = tan x



-360^0 360^0

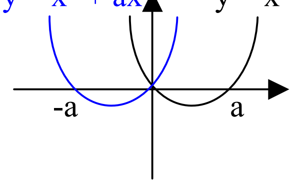


y = cos x

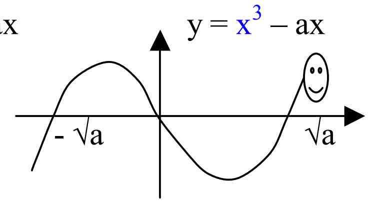
The cosine function is an even function: $\cos(-x) = \cos(x)$

Some transformed curves:

$y = x^2 + ax$ $y = x^2 - ax$



$y = x^3 - ax$



$y = -x^3 + ax$