GRAPHICAL SOLUTION OF EQUATIONS

The first part of questions of this type, where you are asked to draw the graph of a certain function, involves most of the hard work.
The question will first ask you to complete a table of values of ***y*** for certain values of ***x***.

Complete this table for values of $y = x^3 - 2x + 2$. See? I told you so!

x =	**- 2.5**	**- 2**	**- 1**	**0**	**1**	**2**	**2.5**
$y = x^3 - 2x + 2$	**- 8.625**		**3**			**6**	

If you can complete the table by putting in the y values with confidence then go right on ahead!

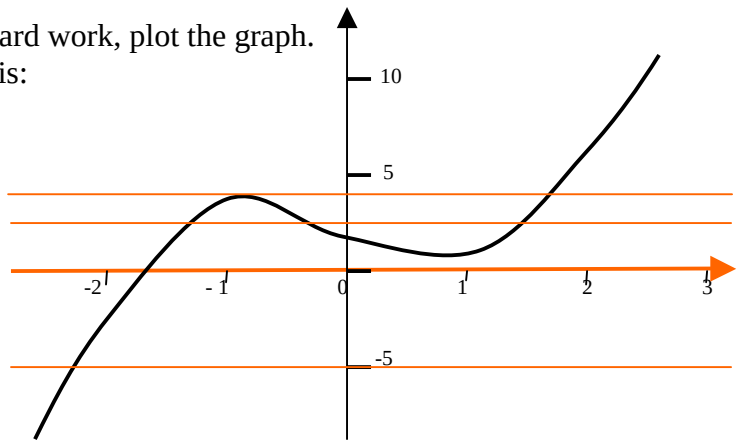
It is usually safer to split the components of the function and concentrate on one aspect at a time, working horizontally.

x =	- 2.5	- 2	- 1	0	1	2	2.5
x^3	**- 15.625**	**- 8**	**- 1**	**0**	**1**	**8**	**15.625**
– 2x	**5**	**4**	**2**	**0**	**- 2**	**- 4**	**- 5**
+ 2	**+ 2**	**+ 2**	**+ 2**	**+ 2**	**+ 2**	**+ 2**	**+ 2**

We may then add vertically to obtain values for y.

$y = x^3 - 2x + 2$	- 8.625	- 2	3	2	1	6	12.625

To complete the hard work, plot the graph.
It will look like this:



solves $x^3 - 2x + 2 = 3$
solves $x^3 - 2x + 2 = 2.5$
solves $x^3 - 2x + 2 = 0$
solves $x^3 - 2x + 2 = - 5$

This graph can now be used to solve a multitude of equations.
The idea is that the x-values at the intersection of the two graphs is the solution of simultaneous equations.

To solve (i) $x^3 - 2x + 2 = 2.5$ or (ii) $x^3 - 2x + 2 = 0$ or (iii) $x^3 - 2x + 2 = - 5$
Notice the left hand side of the equation has already been drawn.
Draw the right hand side and read off the solution(s).

Draw **y = 2.5** **y = 0** **y = - 5**
Solutions (i) x = - 1.3, - 0.4, 1.5 (ii) x = - 1.7 (iii) x = - 2.3

We have solved simultaneously: $y = x^3 - 2x + 2$, $y = 2.5$ | $y = x^3 - 2x + 2$, $y = 0$ | $y = x^3 - 2x + 2$, $y = -5$

Solutions of $x^3 - 2x + 2 = 3$ may be obtained by drawing the line $y = 3$ and finding the solutions.

$x = -1, -1, 1.8$

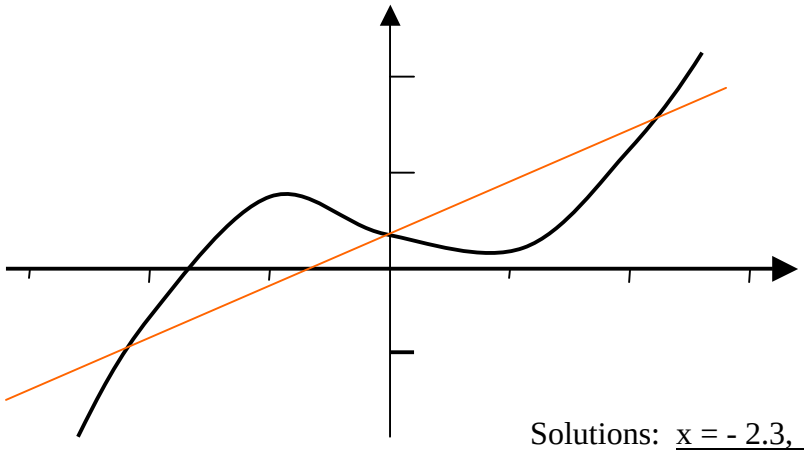
Notice that one solution is repeated. I.e. there are only two distinct solutions.

To solve an equation like $x^3 - 2x + 2 = 3x + 2$ simply draw the line $y = 3x + 2$ since the function $3x + 2$ is the right hand side.

To solve $x^3 - 5x = 0$, using the curve $y = x^3 - 2x + 2$, we must rearrange the left hand side to fit the existing curve:

Write $x^3 - 2x - 3x + 2 - 2 = 0$ and rearrange: $x^3 - 2x + 2 = +3x + 2$

Now draw on the line $y = 3x + 2$ and read off the solutions.



Solutions: $x = -2.3,\ 0,\ 2.3$

Existing curve	Equation to be solved	Rearrangements	Line to draw
$y = 2x^2 - 4x - 3$	$2x^2 - 4x - 7 = 0$	$2x^2 - 4x - 3 - 4 = 0$ $2x^2 - 4x - 3 = 4$	$y = 4$
$y = 3x - \frac{5}{x}$	$3x - \frac{5}{x} - 10 = 0$	$3x - \frac{5}{x} = 10$	$y = 10$
$y = 2x^2 - x - 4$	$2x^2 - x = x + 5$	$2x^2 - x - 4 = x + 5 - 4$ $2x^2 - x - 4 = x + 1$	$y = x + 1$
$y = 6x - x^2$	$5x - 3 - x^2 = 0$	$6x - x - 3 - x^2 = 0$ $6x - x^2 = x + 3$	$y = x + 3$

If we are given a curve and a line and wish to find the **equation** whose solutions can be found from reading the intersections, we eliminate y, put the two functions equal and tidy up.

Existing curve	Line	Eliminate y	Equation solved
$y = 4 + 3x - x^2$	$y = 3$	$4 + 3x - x^2 = 3$	$1 + 3x - x^2 = 0$
$y = 2x + \frac{8}{x}$	This is actually another curve: $y = 12 - \frac{x}{2}$	$2x + \frac{8}{x} = 12 - \frac{x}{2}$	multiply through by 2x $4x^2 + 16 = 24x - x^2$ $5x^2 - 24x + 16 = 0$