# GRAPHICAL SOLUTION OF EQUATIONS

The first part of questions of this type, where you are asked to draw the graph of a certain function, involves most of the hard work.
The question will first ask you to complete a table of values of ***y*** for certain values of ***x***.

**Complete this table for values of $y = x^3 - 2x + 2$.** See? I told you so!

| **x =** | **- 2.5** | **- 2** | **- 1** | **0** | **1** | **2** | **2.5** |
|---|---|---|---|---|---|---|---|
| **$y = x^3 - 2x + 2$** | **- 8.625** | | **3** | | | **6** | |

If you can complete the table by putting in the y values with confidence then go right on ahead!

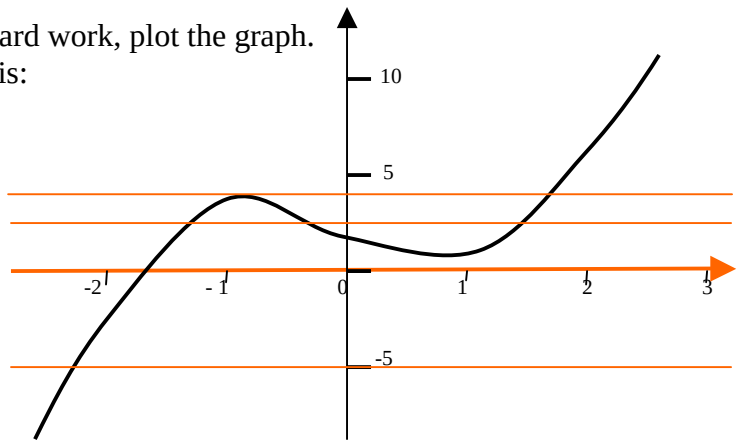
It is usually safer to split the components of the function and concentrate on one aspect at a time, working horizontally.

| x = | - 2.5 | - 2 | - 1 | 0 | 1 | 2 | 2.5 |
|---|---|---|---|---|---|---|---|
| **$x^3$** | **- 15.625** | **- 8** | **- 1** | **0** | **1** | **8** | **15.625** |
| **– 2x** | **5** | **4** | **2** | **0** | **- 2** | **- 4** | **- 5** |
| **+ 2** | **+ 2** | **+ 2** | **+ 2** | **+ 2** | **+ 2** | **+ 2** | **+ 2** |

We may then add vertically to obtain values for y.

| $y = x^3 - 2x + 2$ | - 8.625 | - 2 | 3 | 2 | 1 | 6 | 12.625 |
|---|---|---|---|---|---|---|---|

To complete the hard work, plot the graph.
It will look like this:




solves $x^3 - 2x + 2 = 3$
solves $x^3 - 2x + 2 = 2.5$
solves $x^3 - 2x + 2 = 0$
solves $x^3 - 2x + 2 = - 5$

This graph can now be used to solve a multitude of equations.
The idea is that the x-values at the intersection of the two graphs is the solution of simultaneous equations.

To solve (i) $x^3 - 2x + 2 = 2.5$ or (ii) $x^3 - 2x + 2 = 0$ or (iii) $x^3 - 2x + 2 = - 5$
Notice the left hand side of the equation has already been drawn.
Draw the right hand side and read off the solution(s).

Draw **y = 2.5** **y = 0** **y = - 5**
Solutions (i) x = - 1.3, - 0.4, 1.5 (ii) x = - 1.7 (iii) x = - 2.3

We have solved simultaneously: $y = x^3 - 2x + 2$, $y = 2.5$ | $y = x^3 - 2x + 2$, $y = 0$ | $y = x^3 - 2x + 2$, $y = -5$

Solutions of $x^3 - 2x + 2 = 3$ may be obtained by drawing the line $y = 3$ and finding the solutions.

$x = -1, -1, 1.8$

Notice that one solution is repeated. I.e. there are only two distinct solutions.

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To solve an equation like $x^3 - 2x + 2 = 3x + 2$ simply draw the line $y = 3x + 2$ since the function $3x + 2$ is the right hand side.

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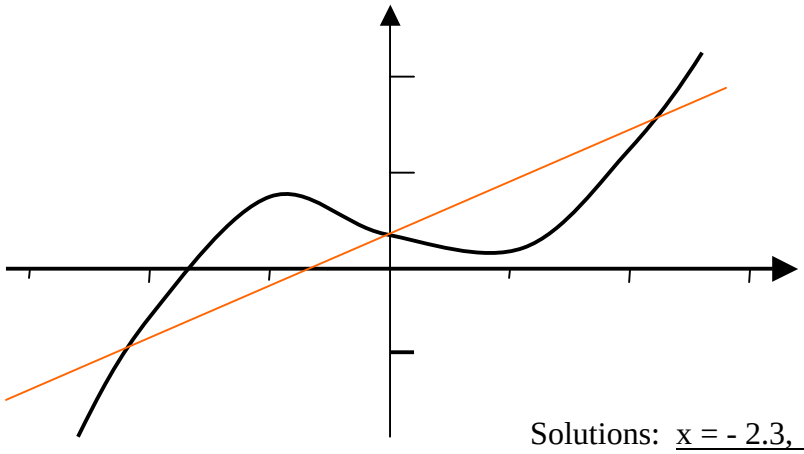
To solve $x^3 - 5x = 0$, using the curve $y = x^3 - 2x + 2$, we must rearrange the left hand side to fit the existing curve:

Write $x^3 - 2x - 3x + 2 - 2 = 0$ and rearrange: $x^3 - 2x + 2 = +3x + 2$

Now draw on the line $y = 3x + 2$ and read off the solutions.



Solutions: $x = -2.3,\ 0,\ 2.3$

| Existing curve | Equation to be solved | Rearrangements | Line to draw |
|---|---|---|---|
| $y = 2x^2 - 4x - 3$ | $2x^2 - 4x - 7 = 0$ | $2x^2 - 4x - 3 - 4 = 0$<br>$2x^2 - 4x - 3 = 4$ | $y = 4$ |
| $y = 3x - \frac{5}{x}$ | $3x - \frac{5}{x} - 10 = 0$ | $3x - \frac{5}{x} = 10$ | $y = 10$ |
| $y = 2x^2 - x - 4$ | $2x^2 - x = x + 5$ | $2x^2 - x - 4 = x + 5 - 4$<br>$2x^2 - x - 4 = x + 1$ | $y = x + 1$ |
| $y = 6x - x^2$ | $5x - 3 - x^2 = 0$ | $6x - x - 3 - x^2 = 0$<br>$6x - x^2 = x + 3$ | $y = x + 3$ |

If we are given a curve and a line and wish to find the **equation** whose solutions can be found from reading the intersections, we eliminate y, put the two functions equal and tidy up.

| Existing curve | Line | Eliminate y | Equation solved |
|---|---|---|---|
| $y = 4 + 3x - x^2$ | $y = 3$ | $4 + 3x - x^2 = 3$ | $1 + 3x - x^2 = 0$ |
| $y = 2x + \frac{8}{x}$ | This is actually another curve:<br>$y = 12 - \frac{x}{2}$ | $2x + \frac{8}{x} = 12 - \frac{x}{2}$ | multiply through by 2x<br>$4x^2 + 16 = 24x - x^2$<br>$5x^2 - 24x + 16 = 0$ |