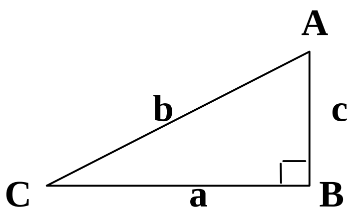
TRIG IDENTITIES

You must learn the formulas. You can get by using the formula booklet but you will waste time in exams unless you can recall the formulas instantly and apply them correctly.

A lot of problems will lead you back to this typical triangle.



Sin, **cos**, and **tan** you know already.

Applying SOHCAHTOA to angle A:

sinA = $\frac{a}{b}$ cosA = $\frac{c}{b}$ tanA = $\frac{a}{c}$

Applying SOHCAHTOA to angle C:

sinC = $\frac{c}{b}$ cosC = $\frac{a}{b}$ tanC = $\frac{c}{a}$

Putting these ratios and the three new ones in one table:

	SIN	COS	TAN	SEC SECANT $= \frac{1}{COS}$	COSEC COSECANT $= \frac{1}{SIN}$	COT COTANGENT $= \frac{1}{TAN}$
ANGLE A	$\frac{a}{b}$	$\frac{c}{b}$	$\frac{a}{c}$	$\frac{b}{c}$	$\frac{b}{a}$	$\frac{c}{a}$
ANGLE C	$\frac{c}{b}$	$\frac{a}{b}$	$\frac{c}{a}$	$\frac{b}{a}$	$\frac{b}{c}$	$\frac{a}{c}$

Notes:

$$\frac{\sin A}{\cos A} = \frac{\frac{a}{b}}{\frac{c}{b}} = \frac{a}{b} \times \frac{b}{c} = \frac{a}{c} = \tan A \qquad \tan A = \frac{\sin A}{\cos A}$$

sinA = cosC = $\frac{a}{b}$, but A and C are complimentary (add to 90^0)

SinA = cos(90 – A), tanA = cot(90 – A), secA = cosec(90 – A)

Examples sin30 = cos60, tan10 = cot 80, sec45 = cosec 45

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sinA = $\frac{a}{b}$ $(\sin A)^2 = (\frac{a}{b})^2$ but we write $\sin^2 A = \frac{a^2}{b^2}$

similarly $\cos^2 A = \frac{c^2}{b^2}$

and $\sin^2 A + \cos^2 A = \dfrac{a^2}{b^2} + \dfrac{c^2}{b^2} = \dfrac{a^2 + c^2}{b^2} = \dfrac{b^2}{b^2} = 1$

$a^2 + c^2 = b^2$
by Pythagoras

$$\mathbf{\sin^2 A + \cos^2 A = 1}$$

Try it with any angle: $\mathbf{\sin^2 52 + \cos^2 52 = 0.62096 + 0.37904 = 1}$

Starting with	$\mathbf{\sin^2 A + \cos^2 A = 1}$
Dividing by $\mathbf{\sin^2 A}$	$\mathbf{1 + \cot^2 A = \text{cosec}^2 A}$
Dividing by $\mathbf{\cos^2 A}$	$\mathbf{\tan^2 A + 1 = \sec^2 A}$

For any angle:

$\mathbf{\sin^2 A + \cos^2 A = 1}$

$\mathbf{\sec^2 A = 1 + \tan^2 A}$

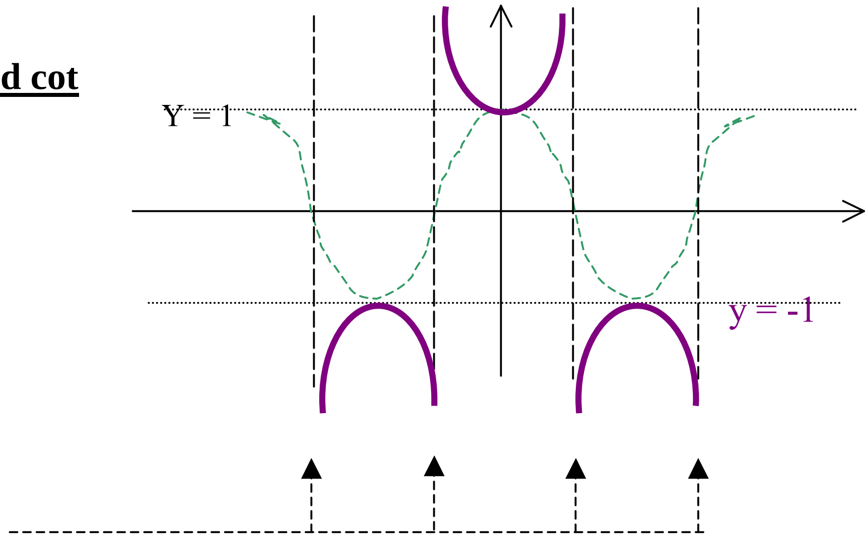
$\mathbf{\text{cosec}^2 A = 1 + \cot^2 A}$

The graphs of sec, cosec and cot

To sketch the graph of **y = secx**

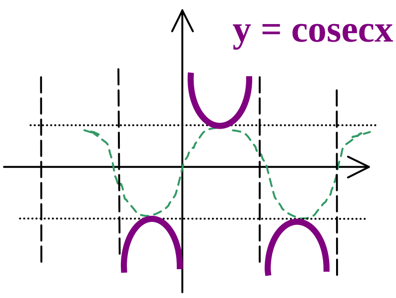
Start with the graph of cosx and -

Work out values of $\dfrac{1}{COSX}$

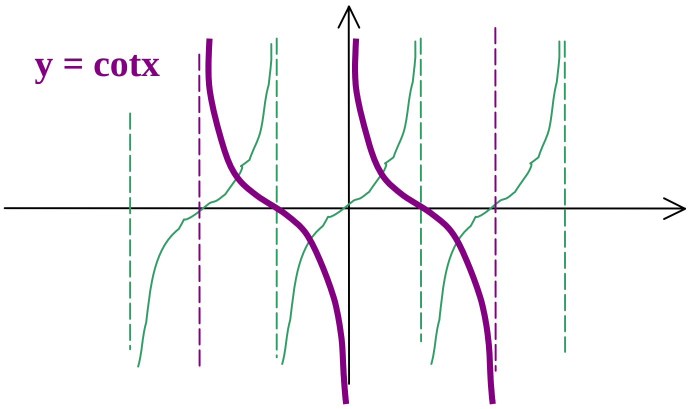


$\dfrac{1}{0} = \infty$ (becomes infinity)
so there will be asymptotes here

y = cosecx



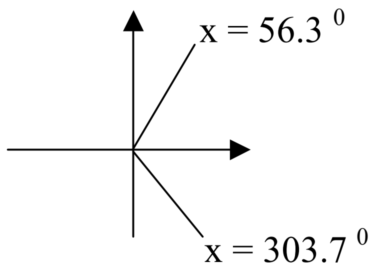
y = cotx



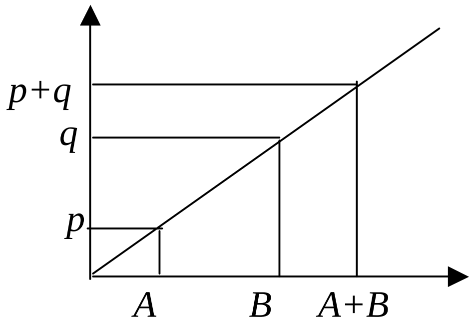
Solving equations: solve $\sec x = 1.8$

Write $\dfrac{1}{COSX} = 1.8$

Cross multiply $\dfrac{1}{1.8} = \cos x$



The Compound angle formula When we compound two angles, we combine them as in a compound (in chemistry).



$\text{Sin}(A + B) \neq \sin A + \sin B$

$\text{Sin}(A + B) = \sin A + \sin B$ would only *be true if the sine graph were a straight line*

Unfortunately the formula connecting the sine of the added angles with their individual sines is a little more complicated. I also involves cosines.

$$\text{Sin}(A + B) = \sin A\cos B + \cos A\sin B$$
$$\text{Sin}(A - B) = \sin A\cos B - \cos A\sin B$$
$$\text{Cos}(A + B) = \cos A\cos B - \sin A\sin B$$
$$\text{Cos}(A - B) = \cos A\cos B + \sin A\sin B$$

$$\text{Tan}(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$
$$\text{Tan}(A - B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

The Double angle formula Replacing B with A:

$\text{Sin}(A + A) = \mathbf{sin2A} = \sin A\cos A + \cos A\sin A = \mathbf{2\,sinAcosA}$
$\text{Cos}(A + A) = \mathbf{cos2A} = \cos A\cos A - \sin A\sin A = \mathbf{cos^2A - sin^2A}$
(Using $\sin^2 + \cos^2 = 1$) $= \mathbf{1 - 2\,sin^2A}$
$= \mathbf{2\,cos^2A - 1}$

$\text{Tan}(A + A) = \mathbf{tan2A} = \dfrac{\tan A + \tan A}{1 - \tan A\tan A} = \dfrac{2\tan A}{1 - \tan^2 A}$

$\text{Tan2A} = \dfrac{\sin A}{\cos A}$

We can even generate a treble angle formula by writing $\sin 3A = \sin(A + 2A)$, Expand this and use the double angle formula to get:
sin3A = 3sinA – 4 sin³A
check this identity and find the cos and tan equivalents.

The Half angle formula

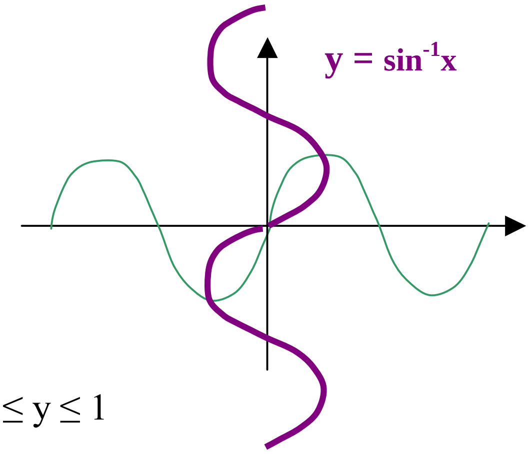
I the double angle formula were to be looked as a double/single link, we could have:

Sin4A = 2 sin2Acos2A or cos10A = cos²5A – sin²5A ……………………..

And then:

$$\mathbf{SinA = 2\sin\tfrac{1}{2}A\cos\tfrac{1}{2}A}$$
$$\mathbf{CosA = \cos^2\tfrac{1}{2}A - \sin^2\tfrac{1}{2}A}$$
$$\mathbf{Tan2A = \frac{2\tan\frac{1}{2}A}{1 - \tan2\frac{1}{2}A}}$$

Inverse Trig functions

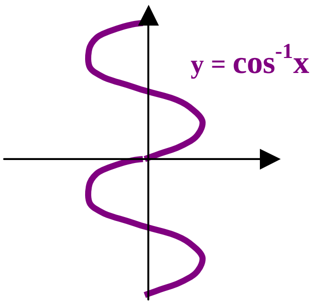


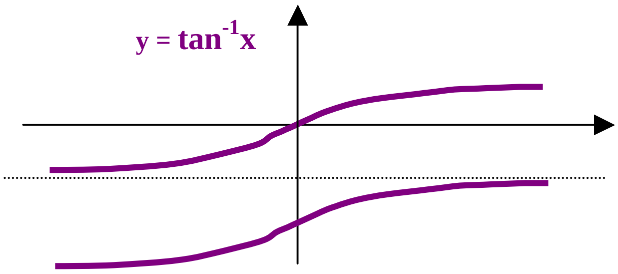
To find the inverse of the function sinx: let $y = \sin x$
Rearrange making x the subject: $x = \sin^{-1} y$
Interchange: $y = \sin^{-1} x$

$f(x) = \sin x$, $f^{-1}(x) = \sin^{-1} x$ this function is the reflection of sinx in the line $y = x$ and may be written $f^{-1}(x) = \arcsin x$

Only a one-to one function can have an inverse so the domain of sinx is defined as $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, range $-1 \le y \le 1$

the inverse function $\sin^{-1}x$ has range $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ and range $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$,





The Factor formula

Two sines or coses may be converted from a sum to a product.

$$\sin P + \sin Q = 2\sin\frac{P+Q}{2}\cos\frac{P-Q}{2}$$

$$\sin P - \sin Q = 2\cos\frac{P+Q}{2}\sin\frac{P-Q}{2}$$

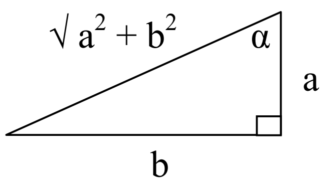
$$\cos P + \cos Q = 2\cos\frac{P+Q}{2}\cos\frac{P-Q}{2}$$

$$\cos P - \cos Q = -2\sin\frac{P+Q}{2}\sin\frac{P-Q}{2}$$

Alternatively:

$\sin(A+B) + \sin(A-B) = 2\sin A\cos B$
$\sin(A+B) - \sin(A-B) = 2\cos A\cos B$
$\cos(A+B) + \cos(A-B) = 2\cos A\cos B$
$\cos(A+B) - \cos(A-B) = -2\sin A\sin B$

Solving equations of the form a sinx + b cosx = c



First construct a right-angled triangle with sides a and b.
Then work out the hypotenuse.

Divide the whole equation by the hypotenuse $\frac{a\sin x}{\sqrt{a^2+b^2}} + \frac{b\cos x}{\sqrt{a^2+b^2}} = \frac{c}{\sqrt{a^2+b^2}}$

Introduce α into the triangle and replace the fractions with their trig equivalents

$$\cos\alpha\sin x + \sin\alpha\cos x = \frac{c}{\sqrt{a^2+b^2}}$$

Use the Compound angle formula $\sin(x+\alpha) = \frac{c}{\sqrt{a^2+b^2}}$

Solve this equation bearing in mind that α is known to be $\tan^{-1}\frac{b}{a}$.

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