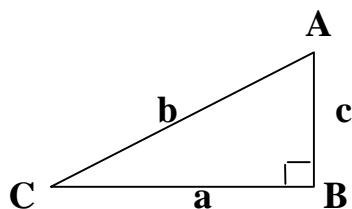


TRIG IDENTITIES

You must learn the formulas. You can get by using the formula booklet but you will waste time in exams unless you can recall the formulas instantly and apply them correctly.

A lot of problems will lead you back to this typical triangle.



Sin, cos, and tan you know already.

Applying SOHCAHTOA to angle A:

$$\sin A = \frac{a}{b} \qquad \cos A = \frac{c}{b} \qquad \tan A = \frac{a}{c}$$

Applying SOHCAHTOA to angle C:

$$\sin C = \frac{c}{b} \qquad \cos C = \frac{a}{b} \qquad \tan C = \frac{c}{a}$$

Putting these ratios and the three new ones in one table:

	SIN	COS	TAN	SEC SECANT = $\frac{1}{\text{COS}}$	COSEC COSECANT = $\frac{1}{\text{SIN}}$	COT COTANGENT = $\frac{1}{\text{TAN}}$
ANGLE A	$\frac{a}{b}$	$\frac{c}{b}$	$\frac{a}{c}$	$\frac{b}{c}$	$\frac{b}{a}$	$\frac{c}{a}$
ANGLE C	$\frac{c}{b}$	$\frac{a}{b}$	$\frac{c}{a}$	$\frac{b}{a}$	$\frac{b}{c}$	$\frac{a}{c}$

Notes:

$$\frac{\sin A}{\cos A} = \frac{\frac{a}{b}}{\frac{c}{b}} = \frac{a}{b} \times \frac{b}{c} = \frac{a}{c} = \tan A \qquad \tan A = \frac{\sin A}{\cos A}$$

$\sin A = \cos C = \frac{a}{b}$, but A and C are complimentary (add to 90°)

$\sin A = \cos(90 - A)$, $\tan A = \cot(90 - A)$, $\sec A = \text{cosec}(90 - A)$

Examples $\sin 30 = \cos 60$, $\tan 10 = \cot 80$, $\sec 45 = \text{cosec } 45$

$\sin A = \frac{a}{b}$ $(\sin A)^2 = \left(\frac{a}{b}\right)^2$ but we write $\sin^2 A = \frac{a^2}{b^2}$

similarly $\cos^2 A = \frac{c^2}{b^2}$

and $\sin^2 A + \cos^2 A = \frac{a^2}{b^2} + \frac{c^2}{b^2} = \frac{a^2 + c^2}{b^2} = \frac{b^2}{b^2} = 1$

$a^2 + c^2 = b^2$
by Pythagoras

$\sin^2 A + \cos^2 A = 1$

Try it with any angle: $\sin^2 52 + \cos^2 52 = 0.62096 + 0.37904 = 1$

Starting with
Dividing by $\sin^2 A$
Dividing by $\cos^2 A$

$\sin^2 A + \cos^2 A = 1$
 $1 + \cot^2 A = \operatorname{cosec}^2 A$
 $\tan^2 A + 1 = \sec^2 A$

For any angle:
 $\sin^2 A + \cos^2 A = 1$
 $\sec^2 A = 1 + \tan^2 A$
 $\operatorname{cosec}^2 A = 1 + \cot^2 A$

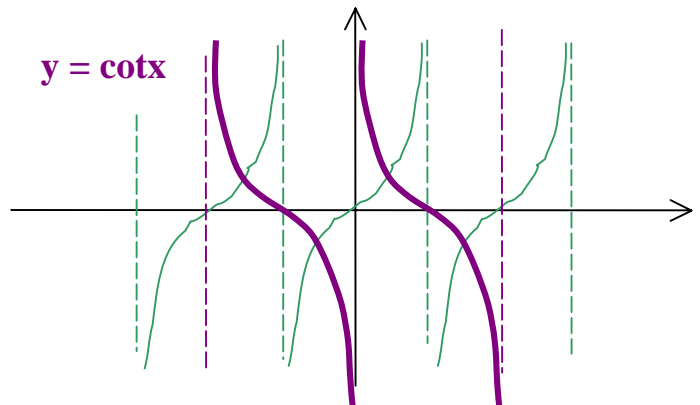
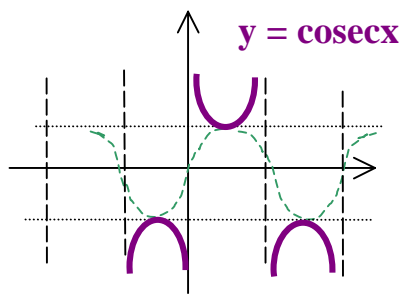
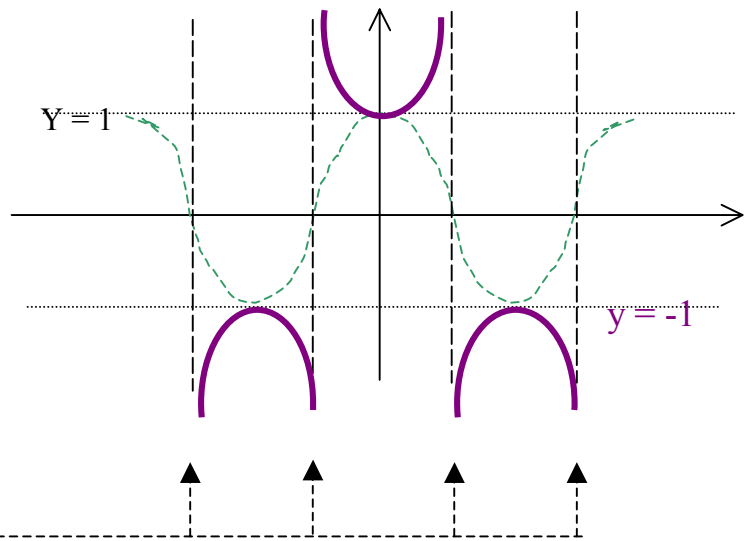
The graphs of sec, cosec and cot

To sketch the graph of $y = \sec x$

Start with the graph of $\cos x$ and -

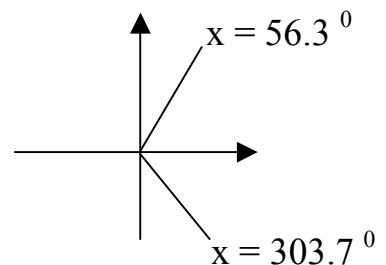
Work out values of $\frac{1}{\cos x}$

$\frac{1}{0} = \infty$ (becomes infinity)
so there will be asymptotes here



Solving equations: solve
Write
Cross multiply

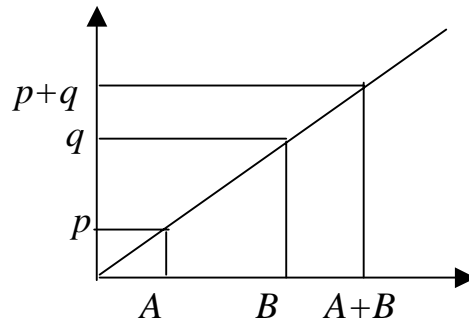
$\sec x = 1.8$
 $\frac{1}{\cos x} = 1.8$
 $\frac{1}{1.8} = \cos x$



The Compound angle formula When we compound two angles, we combine them as in a compound (in chemistry).

$$\sin(A + B) \neq \sin A + \sin B$$

$\sin(A + B) = \sin A + \sin B$ would only be true if the sine graph were a straight line



Unfortunately the formula connecting the sine of the added angles with their individual sines is a little more complicated. It also involves cosines.

$$\begin{aligned} \sin(A + B) &= \sin A \cos B + \cos A \sin B \\ \sin(A - B) &= \sin A \cos B - \cos A \sin B \\ \cos(A + B) &= \cos A \cos B - \sin A \sin B \\ \cos(A - B) &= \cos A \cos B + \sin A \sin B \end{aligned}$$

$$\begin{aligned} \tan(A + B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B} \\ \tan(A - B) &= \frac{\tan A - \tan B}{1 + \tan A \tan B} \end{aligned}$$

The Double angle formula

Replacing B with A:

$$\begin{aligned} \sin(A + A) &= \sin 2A = \sin A \cos A + \cos A \sin A = 2 \sin A \cos A \\ \cos(A + A) &= \cos 2A = \cos A \cos A - \sin A \sin A = \cos^2 A - \sin^2 A \\ &\quad (\text{Using } \sin^2 + \cos^2 = 1) = 1 - 2 \sin^2 A \\ &\quad = 2 \cos^2 A - 1 \\ \tan(A + A) &= \tan 2A = \frac{\tan A + \tan A}{1 - \tan A \tan A} = \frac{2 \tan A}{1 - \tan^2 A} \\ \tan 2A &= \frac{\sin A}{\cos A} \end{aligned}$$

We can even generate a treble angle formula by writing $\sin 3A = \sin(A + 2A)$, Expand this and use the double angle formula to get:

$\sin 3A = 3 \sin A - 4 \sin^3 A$
[check this identity and find the cos and tan equivalents.](#)

The Half angle formula

If the double angle formula were to be looked as a double/single link, we could have:

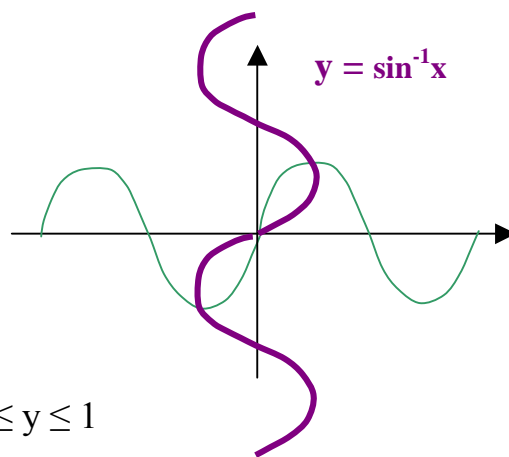
$\sin 4A = 2 \sin 2A \cos 2A$ or $\cos 10A = \cos^2 5A - \sin^2 5A$

And then:

$$\begin{aligned} \sin A &= 2 \sin \frac{1}{2} A \cos \frac{1}{2} A \\ \cos A &= \cos^2 \frac{1}{2} A - \sin^2 \frac{1}{2} A \\ \tan 2A &= \frac{2 \tan \frac{1}{2} A}{1 - \tan^2 \frac{1}{2} A} \end{aligned}$$

Inverse Trig functions

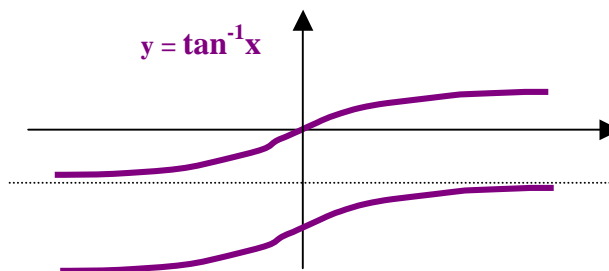
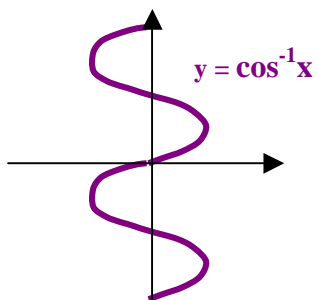
To find the inverse of the function $\sin x$: let $y = \sin x$
 Rearrange making x the subject: $x = \sin^{-1}y$
 Interchange: $y = \sin^{-1}x$



$f(x) = \sin x$, $f^{-1}(x) = \sin^{-1}x$ this function is the reflection of $\sin x$ in the line $y = x$ and may be written $f^{-1}(x) = \arcsin x$

Only a one-to-one function can have an inverse so the domain of $\sin x$ is defined as $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$, range $-1 \leq y \leq 1$

the inverse function $\sin^{-1}x$ has range $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$ and range $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$,



The Factor formula

Two sines or coses may be converted from a sum to a product.

$$\sin P + \sin Q = 2\sin \frac{P+Q}{2} \cos \frac{P-Q}{2}$$

$$\sin P - \sin Q = 2\cos \frac{P+Q}{2} \sin \frac{P-Q}{2}$$

$$\cos P + \cos Q = 2\cos \frac{P+Q}{2} \cos \frac{P-Q}{2}$$

$$\cos P - \cos Q = -2\sin \frac{P+Q}{2} \sin \frac{P-Q}{2}$$

Alternatively:

$$\sin(A+B) + \sin(A-B) = 2\sin A \cos B$$

$$\sin(A+B) - \sin(A-B) = 2\cos A \cos B$$

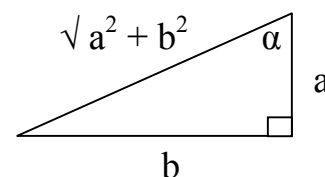
$$\cos(A+B) + \cos(A-B) = 2\cos A \cos B$$

$$\cos(A+B) - \cos(A-B) = -2\sin A \sin B$$

Solving equations of the form $a \sin x + b \cos x = c$

First construct a right-angled triangle with sides a and b .

Then work out the hypotenuse.



Divide the whole equation by the hypotenuse

$$\frac{a \sin x}{\sqrt{a^2 + b^2}} + \frac{b \cos x}{\sqrt{a^2 + b^2}} = \frac{c}{\sqrt{a^2 + b^2}}$$

Introduce α into the triangle and replace the fractions with their trig equivalents

$$\cos \alpha \sin x + \sin \alpha \cos x = \frac{c}{\sqrt{a^2 + b^2}}$$

Use the Compound angle formula $\sin(x + \alpha) = \frac{c}{\sqrt{a^2 + b^2}}$

Solve this equation bearing in mind that α is known to be $\tan^{-1} \frac{b}{a}$.