INDICE^{S}

MATHEMATICAL SHORTHAND

Our first introduction to indices comes when we consider square numbers like 3^2.

We then progress to numbers like 3^5 which is the index form of $3 \times 3 \times 3 \times 3 \times 3$

Now that we are talking in terms of index form, we need to learn how these numbers work when they are manipulated.

THE LAWS OF INDICES

1. $a^x \times a^y = a^{(x + y)}$

Multiplication:

Notice that $3^5 \times 3^3 = (3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3 \times 3) = 3^8$

We have **added** the indices.

When multiplying, **add** the indices.

2. $a^x \div a^y = a^{(x - y)}$

Division:

Notice that $3^5 \div 3^3 = (3x3x3x3x3) \div (3x3x3) = \frac{3x3x3x3x3}{3x3x3} = 3^2$ after cancelling.

We have **subtracted** the indices.

When dividing, **subtract** the indices.

3. $(a^x)^y = a^{x y}$

Power of a power:

Notice that $(3^5)^2 = (3x3x3x3x3) \times (3x3x3x3x3) = 3^{10}$

We have **multiplied** the indices.

When taking an index of an index, **multiply** the indices.

Now, if you remember back to your first teddy and how you learnt to count:- The first numbers you learnt were the positive integers 1, 2, 3….

You understand all these when applied as indices: 3^1, 3^2, 3^3,……

4. $a^0 = 1$

Zero index:

You then learnt the number zero when you had no teddies.

How does zero work as an index? To find out let us use it in a calculation.

$a^x \times a^0 = a^{(x + 0)} = a^x$, (when multiplying add the indices)

$a^0 = \frac{a^x}{a^x} = 1$ Any number to the power of zero will be **1**.

The next numbers you learnt were fractions (when you lost bits of your teddies).

Let us use a fractional index in a calculation.

5. $a^{½} = \sqrt{a}$

$a^{\frac{1}{3}} = \sqrt[3]{a}$

$a^{\frac{1}{n}} = \sqrt[n]{a}$

Fractional index $9^{\frac{1}{2}} \times 9^{\frac{1}{2}} = 9^1$, (when multiplying add the indices)

$9^{\frac{1}{2}}$ must be the number which, when multiplied by itself gives 9. ie. 3, which is the square root of 9 . So $9^{\frac{1}{2}} = \sqrt{9}$

6. $a^{-n} = \frac{1}{a^n}$

7. $a^{\frac{p}{q}} = (a^{\frac{1}{q}})^p$

8. $\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$

Then you started growing up and began a bit of wheeling and dealing by lending or swapping teddies. It was here you started learning about negative numbers.

Negative index

Putting a negative number into a calculation:

$3^{-1} \times 3^{1} = 3^{0} = 1$ **cross multiplying gives** $3^{-1} = \frac{1}{3^1}$

So when we have a negative index, bring it to the denominator and make the index positive.

$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$, $3^{-3} = \frac{1}{3^3} = \frac{1}{27}$, $3^{-4} = \frac{1}{3^3} = \frac{1}{81}$

Negative/fractional index

$9^{-\frac{1}{2}} = \frac{1}{9^{\frac{1}{2}}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$ done by a combination of the negative and fraction rules.

Indices of the form $\frac{a}{b}$

$27^{\frac{2}{3}}$ split the fraction into $\frac{1}{3}$ x 2 and use the reverse of rule 3

$(27^{\frac{1}{3}})^2 = (3)^2 = 9$ $27^{-\frac{2}{3}} = \frac{1}{27^{\frac{2}{3}}} = \frac{1}{9}$

Indices of fractions

$\left(\frac{a}{b}\right)^{-1} = \left(\frac{1}{\frac{a}{b}}\right)$ (but when dividing by a fraction we turn the divisor upside down and multiply) $= \frac{b}{a}$

$\left(\frac{a}{b}\right)^{2} = \frac{a^2}{b^2}$, $\left(\frac{a}{b}\right)^{-2} = \frac{b^2}{a^2}$, $\left(\frac{a}{b}\right)^{\frac{1}{2}} = \left(\frac{\sqrt{a}}{\sqrt{b}}\right)$

Say goodbye to Teddy!