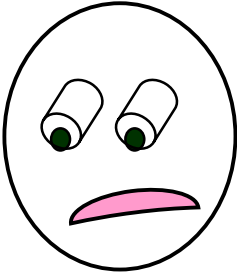


IRRATIONAL NUMBERS



An irrational number is one which cannot be written in the form $\frac{a}{b}$

where a and b are integers (whole numbers).

π is one of the more well known irrational numbers.

A rational approximation often used for π is $\frac{22}{7}$ which is

3.1428571 from a calculator and compares to the true value of $\pi = 3.1415927$ to 7 dp. The match is only to 2 decimal places.

Mathematicians have found various rational approximations but the resulting value will only match up to a certain decimal place.

In about 500 AD the Chinese mathematician Tsu Ch'ung-Chi produced the value as 355/113

This gives us 3.1415929 which matches

3.1415926535897932384626433832795028841971693993751.... to 6 decimal places.

Other irrational numbers continue like this and cannot be written as $\frac{a}{b}$.

The square root of a prime number is irrational: $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}, \sqrt{11}, \sqrt{13}, \dots$

Between 1 and 50, the only square roots which are rational are $\sqrt{1}, \sqrt{4}, \sqrt{9}, \sqrt{16}, \sqrt{25}, \sqrt{36}, \sqrt{49}$

That means there are 43 square roots which are irrational. For instance $\sqrt{12}$ and $\sqrt{48}$.

$\sqrt{12}$ may look rational but it can be simplified to $\sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2 \times \sqrt{3}$

we cannot shift the $\sqrt{3}$. So $\sqrt{12}$, which is simplified to $2\sqrt{3}$ is irrational.

$\sqrt{48} = \sqrt{2 \times 2 \times 2 \times 2 \times 3} = 2 \times 2 \times \sqrt{3} = 4\sqrt{3}$



Every pair of numbers reduces to one and we can't reduce the $\sqrt{3}$

There are actually more irrational numbers than rational numbers in our number system!

The cube root of a number which is not a cube is irrational:

$\sqrt[3]{8}$	$\sqrt[3]{54}$	$\sqrt[3]{216}$	$\sqrt[3]{1080}$
RATIONAL	IRRATIONAL	RATIONAL	IRRATIONAL
$\sqrt[3]{2 \times 2 \times 2} = 2$ EVERY TRIPLET REDUCES TO ONE	$\sqrt[3]{3 \times 3 \times 3 \times 2} = 3\sqrt{2}$	$\sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3} = 2 \times 3 = 6$	$\sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5} = 2 \times 3 \times \sqrt{5}$

RECURRING DECIMALS

Recurring decimals occur when certain fractions are divided out. These recurring decimals are therefore rational numbers.

For example $\frac{5}{11}$ divides out to 0.45454545....., recurring every 2 decimal places.

To convert a recurring decimal into a rational number: Let

$$x = 0.45454545 \dots$$

Then multiplying by 100:

$$100x = 45.45454545 \dots$$

Subtract:

$$100x - x = 45$$

$$99x = 45, \quad x = \frac{45}{99}$$

Using the same method and multiplying by 1000 we may find the fraction equivalent of decimals recurring after every 3 decimal places.

Eg. **0.456456456.....**

$$\text{Let } x = 0.456456456\dots$$

$$1000x = 456.456456\dots$$

$$999x = 456$$

$$x = \frac{456}{999}$$

You should be able to do the following problems in your head:

RECURRING DECIMAL	0.777777....	0.34563456.....	0.53535353.....	1.234343434....
FRACTION EQUIVALENT				Perhaps not!

$$1.2343434\dots = 1 + 0.2 + 0.0343434\dots$$

$$= 1 + \frac{2}{10} +$$

$$\begin{aligned} x &= 0.0343434\dots \\ 100x &= 3.43434\dots \\ 99x &= 3.4 \\ x &= \frac{3.4}{99} = \frac{34}{990} \end{aligned}$$

$$= 1 + \frac{2}{10} + \frac{34}{990}$$

$$= 1 + \frac{198}{990} + \frac{34}{990}$$

$$= 1 \frac{232}{990} \text{ (which should be cancelled down).}$$

COMBINATIONS OF IRRATIONAL NUMBERS

Two or more irrational numbers may be multiplied or divided to give rational or irrational numbers.

$\sqrt{12}$ and $\sqrt{48}$ are both irrational but both have the irrational factor $\sqrt{3}$

$\sqrt{12}$ is $2\sqrt{3}$ and $\sqrt{48}$ is $4\sqrt{3}$. When they are multiplied we get $8\sqrt{3}\sqrt{3} = 24$ (Rational).

The two irrational numbers $\sqrt{3}$ and $\sqrt{7}$, when multiplied give $\sqrt{21}$ (irrational).

Note that: $\sqrt{a} \times \sqrt{a} = a$

Numbers written in the form of a square root are said to be written in **surd form**.

Since there are actually more irrational numbers than rational numbers in our number system, there are many irrational numbers between 5 and 6.

5 is $\sqrt{25}$ and 6 is $\sqrt{36}$ therefore some of the irrational numbers between 5 and 6 are $\sqrt{26}$, $\sqrt{27}$,....., $\sqrt{35}$.

↑ And since we can always place another irrational number between any two irrational numbers.

$\sqrt{26.5}$ is an irrational number between $\sqrt{26}$ and $\sqrt{27}$.

If you look at the two setsquares in your geometry set and call one of the sides 1 unit in length, one of the other sides will have an irrational length.

