Logic

Cheap diamonds are rare
All things rare are expensive
Therefore cheap diamonds are expensive

Let us start with a proposition – a statement that is true or false.
A command or a question is not a proposition.
Let us denote propositions by p, q, r and the negation (not p) by $\neg p$.

Write: p: "The kite is high"
$\neg p$: "The kite is not high" (simply insert "not", don't say the kite is low).

q: "There were no empty seats in the house"
$\neg q$: "There were some empty seats in the house"

Compound statements

A <u>and</u> B is written A∩B in set theory
p <u>and</u> q is denoted by $p \wedge q$ This is the conjunction of two propositions.

Truth Tables

A truth table shows how the values (true or false) of a set of propositions affect the values of other propositions.

Truth values of negation

p	$\neg p$
T	F
F	T

If p: "I eat cheese" is true then
$\neg p$: "I do not eat cheese" must be false.

Truth values of conjunction.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

So the truth value of a conjunction is only true when the individual values are true.

Disjunctions and exclusive disjunctions

A <u>or</u> B is written AUB in set theory and means "either A or B or both".
p <u>or</u> q is denoted by $p \vee q$ in symbolic logic This is the conjunction of two propositions.
Exclusive Disjunction: If we mean "either A or B but not both" then we use the symbol $\underline{\vee}$ ($p \underline{\vee} q$)
Exclusive Disjunction: I will sleep at Tom's house or Bill's house after the party.
Disjunction: You need to have passed Physics or Chemistry to enrol on this course.

p	q	$p \vee q$	$p \underline{\vee} q$	
T	T	T	F	Since <u>both</u> are true, the exclusive disjunction is false
T	F	T	T	Since <u>one</u> is true, the disjunction & exclusive disjunction are both true
F	T	T	T	
F	F	F	F	Only FF will give F for both $p \vee q$ and $p \underline{\vee} q$

Test this table using p: "Sue eats sausages" q: "Ruth eats risotto"

Implications

We denote p implies q as $p \Rightarrow q$ and include the words if/then in our statements.

Example: p: "I put my hand in the fire"
q: "I will get burnt"
$p \Rightarrow q$: "*If* I put my hand in the fire *then* I will get burnt"
"If I put my hand in the fire then I won't get burnt" is false
If I don't put my hand in the fire, that does not change the truth of $p \Rightarrow q$

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

So the truth value of an implication is only false when p is true and q is false.

Converse, inverse and contrapositive

The proposition p	The proposition q	The negation of p	The negation of q	The implication of p and q	The converse of p and q	The inverse of p and q	The contrapositive of p and q
p	q	¬p	¬q	$p \Rightarrow q$	$q \Rightarrow p$ $p \Leftarrow q$	$\neg p \Rightarrow \neg q$	$\neg q \Rightarrow \neg p$ $\neg p \Leftarrow \neg q$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

This stuff looks like it might be a pain in the neck to learn but look at it this way:
The $\Rightarrow$ sign only gives **one false** in each column and that false appears when the propositions or the negations have corresponding

T	F

in the row

Another way of looking at it is by looking at the only arrangements of p and q which lead to **F**

T	F	F
p	q	$p \Rightarrow q$
¬p	¬q	$\neg p \Rightarrow \neg q$
q	p	$q \Rightarrow p$
¬q	¬p	$\neg q \Rightarrow \neg p$

Notice that the **implication** and **contrapositive** have the same truth tables
Also, the **converse** and the **inverse** have the same truth tables.

Equivalence

If two statements have the same truth tables then they are equivalent statements.