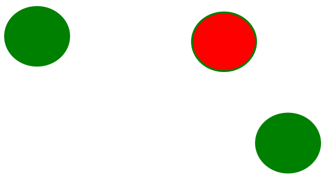
PROBABILITY

The probability of an event happening is written Pr(event) or P(event)

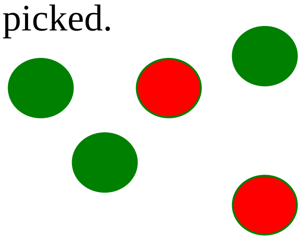
The answer(usually a fraction) is worked out using $\dfrac{\textbf{Number of favourable outcomes}}{\textbf{Total number of outcomes}}$

Example 1: There are 3 red balls and 5 green balls in a bag, A ball is picked at random, find the probability a green ball is picked.



There are 5 green balls $\dfrac{5}{8}$
And 8 balls in total

So Pr(green) = $\dfrac{5}{8}$



Example 2: A 12 sided dice is thrown, find the probability that the number obtained will be a multiple of 3.

There are 4 multiples of three (3, 6, 9, 12) $\dfrac{4}{12}$
There are 12 outcomes in total

So Pr(a multiple of 3) = $\dfrac{4}{12}$ cancel down $\dfrac{1}{3}$

Estimating probabilities and Expected number

To estimate the probability that a drawing pin lands pointy side up:-

Do an experiment - *Drop a drawing pin from the same height 50 times and count the number of times it lands pointy side up - the number of successful outcomes.*

If this happens 26 times then the estimated probability is $\dfrac{26}{50}$ cancel down $\dfrac{13}{25}$

$\dfrac{\text{The number of successful outcomes}}{\text{Total number of trials}}$ is also known as the **relative frequency**.

This is how we may estimate a probability for a drawing pin. We have conducted an experiment.

For the probability that it will snow on Christmas day, we can't throw Christmases, so we look at past data. It snowed 7 times during the last 50 years so

$$\text{Pr(it will snow on Christmas day)} = \frac{7}{50}$$

For the probability of obtaining a number less than 6 on a 12 sided dice we simply look at the **symmetrical outcomes**: There are 12 possible outcomes, all with an equal chance of occurring. There are five possible numbers less than 6.

$$\text{Pr(a number less than 6)} = \frac{5}{12}$$

Estimating probabilities may be done using past data, doing an experiment or looking at symmetry.

The Expected number of times that a drawing pin will fall pointy side up when it is dropped 1000 times.

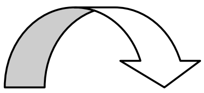
If we already knew or have an estimate of $\frac{13}{25}$ for the probability of a pin landing pointy side up. Simply use that probability and find **that** fraction of 1000.

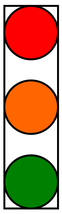
So $\frac{13}{25}$ of 1000 is $\frac{13}{25} \times 1000 = 520$

Expect the pin to fall pointy side up 520 times.

Two or more Trials

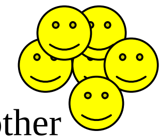
Tossing a coin then tossing it again





Picking a badge, replacing it, then picking another one

Picking a badge, not replacing it, then picking another one



Playing squash against someone then playing tennis

Throwing a dart at a board then throwing another and another

Coming across one set of traffic lights then driving through to another set

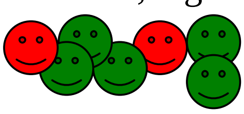
You have to decide if the events are **independent**. Will the outcome of one, effect the outcome of the next event?

There is no memory inside the coin, which tells it that a head has just occurred, so perhaps a tail should show next time. (Silly). Pr(head) will always be $\frac{1}{2}$.

Coin tossing or dart throwing are said to be **independent** if you continue tossing or throwing. **If there is no link between events, they are independent**.

If we **do not replace** a ball before taking another one, then the probability is linked to whatever happened first time. Let us look at this on a **TREE DIAGRAM.**

3 red balls, 5 green balls. Take one, do not replace it, take another one



First ball	Second ball	Code for outcomes	Probability Multiply along branches
$\frac{3}{8}$ red	$\frac{2}{7}$ red	red,red	$\frac{3}{8} \times \frac{2}{7} = \frac{6}{56} = \frac{3}{28}$
	$\frac{5}{7}$ green	red,green	$\frac{3}{8} \times \frac{5}{7} = \frac{15}{56}$
$\frac{5}{8}$ green	$\frac{3}{7}$ red	green,red	$\frac{5}{8} \times \frac{3}{7} = \frac{15}{56}$
	$\frac{4}{7}$ green	green,green	$\frac{5}{8} \times \frac{4}{7} = \frac{20}{56} = \frac{10}{28}$

There are now 7 balls in total as the first one was not put back. Second probability DEPENDS on first

The answers here add up to $\frac{56}{56}$ which is equal to 1.

Find the probability that both balls were red. Answer from top of the tree: $\frac{13}{25}$

Find the probability that both were green. Answer from bottom of tree: $\frac{13}{25}$

It is worth filling in the whole of the Probability Tree before you come to answer the questions.

Find the probability that only one ball is red - This must mean that we obtained:
A red AND green OR a green AND then a red.

AND means you have to multiply OR means you have to add

The answers in the two shaded regions in the table have to be added.

$$\frac{15}{56} + \frac{15}{56} = \frac{30}{56} = \frac{15}{28}$$

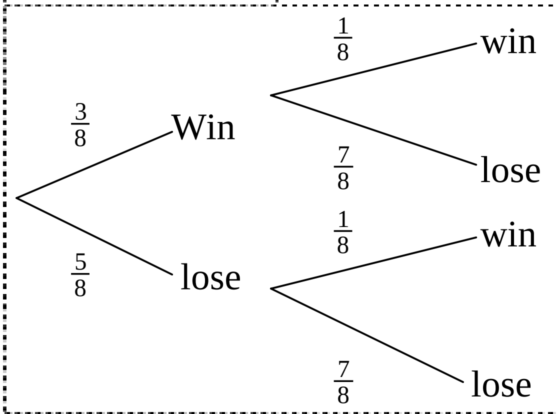
If the first ball is replaced before the second is drawn then the events are independent and the probabilities will be unchanged for the second ball.

First ball	Second ball	Code for outcomes	Probability Multiply along branches
$\frac{3}{8}$ Red	$\frac{3}{8}$ red	red,red	$\frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$
	$\frac{5}{8}$ green	red,green	$\frac{3}{8} \times \frac{5}{8} = \frac{15}{64}$
$\frac{5}{8}$ green	$\frac{3}{8}$ red	green,red	$\frac{5}{8} \times \frac{3}{8} = \frac{15}{64}$
	$\frac{5}{8}$ green	green,green	$\frac{5}{8} \times \frac{5}{8} = \frac{25}{64}$

Other problems with independent events are:

The probability that I win a game of tennis is $\frac{3}{8}$

The probability that I win a game of squash is $\frac{1}{8}$ and is independent of the result in squash.
I am feeling fit so I play you at tennis then at squash.



Tennis	Squash	Code for outcomes	Probability Multiply along branches
		Win, win	Pr(two wins) $\frac{3}{8} \times \frac{1}{8} = \frac{3}{64}$
		Win, lose	Pr(win one) $\frac{3}{8} \times \frac{7}{8} = \frac{21}{64}$
		Lose, win	Pr(win one) $\frac{5}{8} \times \frac{1}{8} = \frac{5}{64}$
		Lose, lose	$\frac{5}{8} \times \frac{7}{8} = \frac{35}{64}$

There are two ways of winning one out of two games.
I could win at tennis **and** lose at squash **or** I could lose at tennis **and** win at squash.

$$\left(\frac{3}{8} \times \frac{5}{8}\right) + \left(\frac{3}{8} \times \frac{5}{8}\right) = \frac{15}{64}$$

but the working will already have been done if you complete the table, before answering the questions. Just add the two answers in the grey boxes.

$$\frac{21}{64} + \frac{5}{64} = \frac{26}{64}$$

Disaster - Whitewash!!

Pr(I lose at tennis **and** lose at squash) = $\frac{5}{8} \times \frac{7}{8} = \frac{35}{64}$