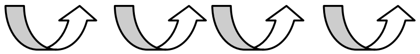
NUMBER PATTERNS

Number patterns have been studied at GCSE level and a formula obtained for the n^{th} term.

TERM	**1** 2	3	4	5	6	
NUMBER	**5**	7	9	11	13	?

The nth term is 2n + 3



GAP 2 2 2 2

The rule for finding the next term is "add 2".

The sequence: 1, 2, 5, 26, 677 also has a rule (or link) for finding the next term from any nth term. $\mathbf{t_{n+1} = t_n^2 + 1}$

At A level you will refer to this as **a recurrence relation** and tells you that
"The next term is obtained by squaring the previous term and adding 1". (We use $\mathbf{t_n}$ **or** $\mathbf{u_n}$ **for the nth term).**

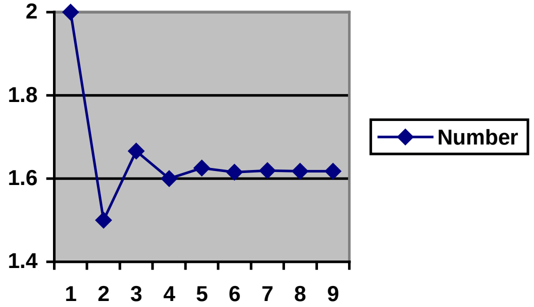
A recurrence relation such as $\mathbf{u_{n+1} = (u_n + 1)^2}$
must also have the first term given or you can't proceed. If you are told ……. $\mathbf{u_1 = 0}$
then you can find the first few terms: **0 1 4 25 676 …….**

<u>Example 1</u> Find the first four terms defined by the following recurrence relation: $\mathbf{u_{n+1} = 2u_n - 1}$
When $u_1 = 1$: **1, 1, 1, 1, 1, not much happening**
When $u_1 = 2$: **2, 3, 5, 9, 17, 33 ….. going up**

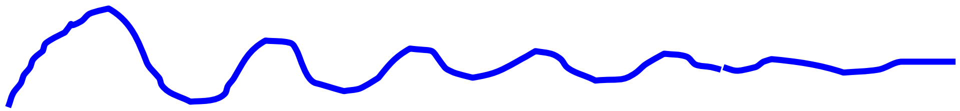
Example 2

The recurrence relation $\mathbf{u_{n+1} = (1/u_n + 1)}$ **with** $\mathbf{u_1 = 1}$ **gives the sequence**

2, 1.5, 1.666, 1.6, 1.625, 1.615, 1.619, 1.618, 1.618, …….. **this sequence is said to be converging**

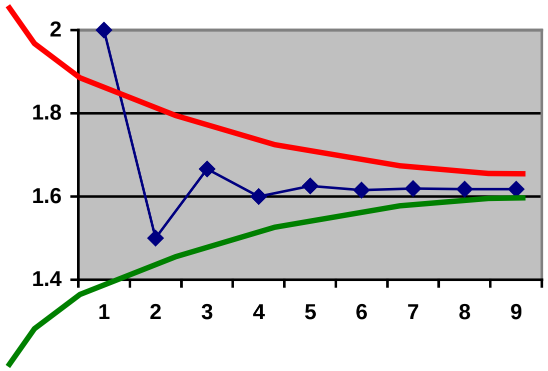


The graph shows how the values settle down and in the long run, they will stabilize around the value 1.6.



If we write $x = \frac{1}{x} + 1$ working with the knowledge that **eventually two consecutive terms will be the same**, we get solutions

$X = \frac{1 \pm \sqrt{5}}{2}$ where $X = \frac{1 + \sqrt{5}}{2}$ is known as the **Golden section.**



Sequences may also converge from **above** or **below**.
They may also **oscillate.**
$\mathbf{u_{n+1} = (2/u_n)}$ $\mathbf{u_1 = -1}$

-1, -2, -1, -2, -1, ………..