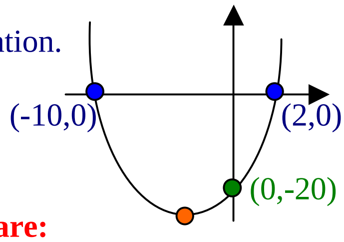
Quadratic Roots

Strictly speaking a Quadratic equation arises from a pair of simultaneous equations:

$y = x^2 + 8x - 20$ is a Quadratic curve and $y = 0$ is the equation of the x axis.
The intersection of the curve and the line is found by eliminating y and getting
$x^2 + 8x - 20 = 0$. This is a Quadratic equation. And when it is solved we get the x-values of the point of intersection with the x-axis:
$x = -10$ and $x = 2$. These two values are known as the **roots** of the equation.



Putting $x = 0$ into $y = x^2 + 8x - 20$ gives us $y = -20$
We can sketch the graph with the relevant points marked.

We can also find the vertex or minimum point by completing the square:
$y = (x + 4)^2 - 36$ and arguing that the lowest value for y must be -36, since a square can never be negative and the value of y can never be lower than -36 whatever the value of x.

Substituting $y = -36$ into the equation of the curve will give us the x-value of the vertex.
$-36 = (x + 4)^2 - 36$ leads to the answer $x = -4$. The vertex is the point $(-4, -36)$

> If a curve can be written in the form $y = (x - a)^2 + b$ the minimum point (occurring when the **square** is zero) is (a,b) and the line of symmetry is $x = a$.

Quadratic equations can be solved using the quadratic formula, but only after you have tried to factorise.

Solving $x^2 + 8x + 30 = 0$. This does not factorise so use the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-8 \pm \sqrt{8^2 - 4\times1\times30}}{2} = \frac{-8 \pm \sqrt{64 - 120}}{2} = \frac{-8 \pm \sqrt{-56}}{2}$$

We cannot take the square root of a negative number so there are no roots.
We say that there are no **real** roots because if we delve into some further maths we can find imaginary roots. We won't go there because you may start asking about my **imaginary** friend!

More importantly, it means that the curve $y = x^2 + 8x + 30$ does not cross the x-axis.
It is not difficult to see that $(b^2 - 4ac)$ is an indicator of the type of roots that a quadratic has.

$(b^2 - 4ac)$ is known as the **discriminant** and it will reveal the nature of the roots to us.

We have just seen that if $(b^2 - 4ac)$ is negative, there are no real roots $(b^2 - 4ac) < 0$

If $(b^2 - 4ac)$ is equal to zero the roots are $\frac{-b \pm \sqrt{0}}{2a}$ i.e. repeated roots $(b^2 - 4ac) = 0$

If $(b^2 - 4ac)$ is positive then we get real and different roots $(b^2 - 4ac) > 0$

If $(b^2 - 4ac)$ is a perfect square then (we could have factorised) the roots are rational $\frac{-b \pm k}{2a}$