

NUMBER PATTERNS

Think of these number patterns as the amount of pocket money you have in your bank

| | | | | | | | | | |
|------|---|---|---|---|---|---|------------|-----------------------------|-------|
| TERM | 1 | 2 | 3 | 4 | 5 | 6 | difference | <u>Formula for nth term</u> | |
|------|---|---|---|---|---|---|------------|-----------------------------|-------|

| | | | | | | |
|--------|---|---|---|----|----|---|
| NUMBER | 5 | 7 | 9 | 11 | 13 | ? |
|--------|---|---|---|----|----|---|



| | | | | |
|-----|---|---|---|---|
| GAP | 2 | 2 | 2 | 2 |
|-----|---|---|---|---|

The **difference** is found by looking at the GAP between the numbers

THE FIRST PART OF THE FORMULA IS kn where k is the same as the gap
 $2n + \text{something}$
 $2n + c$

The second part of the formula is found by substituting $n = 1$ and working out the extra amount needed to get the **FIRST** term
 $2 \times 1 = 2$
3 more will get us to **5**
The formula is $2n + 3$
 Check when $n = 2$:
 $2 \times 2 + 3 = 7$ the second term

Use the formula to get the 20th term - How much money will you have in your bank after 20 weeks?

$$2 \times 20 + 3 = \mathbf{43} \quad \text{The 20th term is } \underline{43}$$






Working backwards: After how many weeks will your bank hold 65 pounds?

Write

$$\begin{aligned} 2n + 3 &= 65 \\ 2n &= 65 - 3 \\ 2n &= 62 \\ n &= \mathbf{31} \end{aligned}$$

WHAT IF THERE IS NO REGULAR GAP BETWEEN NUMBERS?

| | | | | | | |
|--------|---|----|----|----|----|----|
| TERM | 1 | 2 | 3 | 4 | 5 | 6 |
| NUMBER | 4 | 13 | 26 | 43 | 64 | 89 |

| | | | | | |
|-----|---|---|---|--|---|
| |  |  |  |  |  |
| GAP | 9 | 13 | 17 | 21 | 24 |

| | | | | |
|--------------------|---|---|---|---|
| |  |  |  |  |
| SECOND DIFFERENCES | 4 | 4 | 4 | 4 |

CALL THESE THE **FIRST DIFFERENCES**
 THESE ARE NOT THE SAME
 SO THE FORMULA WILL NOT BE
 $Kn + c$ LOOK AT THE **SECOND DIFFERENCES**

THESE ARE THE SAME SO THE FORMULA WILL
 BE A QUADRATIC FORMULA:
 $an^2 + bn + c$ and we have to find a, b and c

a is easy to find - it is always half the second difference - in this case it is **2**

so the formula is $2n^2 + bn + c$ and we have to find **b** and **c**.

| | | | | | | | |
|---|----------|---|---|---|---|---|---|
| Start with a row of term numbers again | n | 1 | 2 | 3 | 4 | 5 | 6 |
|---|----------|---|---|---|---|---|---|

| | | | | | | | |
|-----------------------------------|--------------------------|---|---|----|----|----|----|
| And work out the values of $2n^2$ | $2n^2$ | 2 | 8 | 18 | 32 | 50 | 72 |
|-----------------------------------|--------------------------|---|---|----|----|----|----|

Now write down the difference between these numbers and **the original numbers**

| | | | | | | |
|--|-----|------|------|------|------|------|
| | (4) | (13) | (26) | (43) | (64) | (89) |
|--|-----|------|------|------|------|------|

| | | | | | | | |
|-------------|---|---|---|----|----|----|---|
| Difference: | 2 | 5 | 8 | 11 | 14 | 17 | this sequence is $bn + c$ |
|-------------|---|---|---|----|----|----|---|

And the formula is $3n - 1$ using the method for a simple sequence described above.

Put the two parts together and what have you got?

$$\underline{2n^2} + \underline{3n - 1}$$

The formula for the nth term.