

NUMBER PATTERNS

Think of these number patterns as the amount of pocket money you have in your bank

TERM	1	2	3	4	5	6	difference	<u>Formula for nth term</u>
------	---	---	---	---	---	---	------------	-----------------------------	-------

NUMBER	5	7	9	11	13	?
--------	---	---	---	----	----	---



GAP	2	2	2	2
-----	---	---	---	---

The **difference** is found by looking at the GAP between the numbers

THE FIRST PART OF THE FORMULA IS kn where k is the same as the gap
 $2n + \text{something}$
 $2n + c$

The second part of the formula is found by substituting $n = 1$ and working out the extra amount needed to get the **FIRST** term
 $2 \times 1 = 2$
3 more will get us to **5**
The formula is $2n + 3$
 Check when $n = 2$:
 $2 \times 2 + 3 = 7$ the second term

Use the formula to get the 20th term - How much money will you have in your bank after 20 weeks?

$$2 \times 20 + 3 = \mathbf{43} \quad \text{The 20th term is } \underline{43}$$






Working backwards: After how many weeks will your bank hold 65 pounds?

Write

$$\begin{aligned} 2n + 3 &= 65 \\ 2n &= 65 - 3 \\ 2n &= 62 \\ n &= \mathbf{31} \end{aligned}$$

WHAT IF THERE IS NO REGULAR GAP BETWEEN NUMBERS?

TERM	1	2	3	4	5	6
NUMBER	4	13	26	43	64	89

					
GAP	9	13	17	21	24

				
SECOND DIFFERENCES	4	4	4	4

<p>CALL THESE THE FIRST DIFFERENCES THESE ARE NOT THE SAME SO THE FORMULA WILL NOT BE $Kn + c$ LOOK AT THE SECOND DIFFERENCES</p>

<p>THESE ARE THE SAME SO THE FORMULA WILL BE A QUADRATIC FORMULA: $an^2 + bn + c$ and we have to find a, b and c</p>

a is easy to find - it is always half the second difference - in this case it is **2**

so the formula is $2n^2 + bn + c$ and we have to find **b** and **c**.

Start with a row of term numbers again	n	1	2	3	4	5	6
---	----------	---	---	---	---	---	---

And work out the values of $2n^2$	$2n^2$	2	8	18	32	50	72
-----------------------------------	--------------------------	---	---	----	----	----	----

Now write down the difference between these numbers and **the original numbers**

	(4)	(13)	(26)	(43)	(64)	(89)
--	-----	------	------	------	------	------

Difference:	2	5	8	11	14	17	this sequence is $bn + c$
-------------	---	---	---	----	----	----	---

And the formula is $3n - 1$ using the method for a simple sequence described above.

Put the two parts together and what have you got?

$2n^2$ + $3n - 1$

The formula for the nth term.