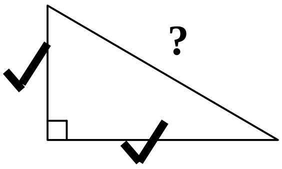
<u>SINE AND COSINE FORMULA</u>

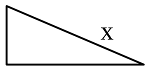
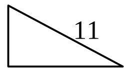
How to solve all kinds of triangles.

To find a missing side when the triangle has a right-angle:

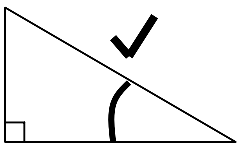


Use Pythagoras' Theorem

(The hypotenuse) ^{2} = (One side) ^{2} + (The other side) ^{2}

Opening line	**x^2** = 3^2 + 4^2  x^2 is on the left	**11^2** = x^2 + 7^2  x^2 is on the right

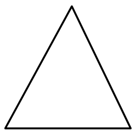
If you have a right-angled triangle and there is another angle involved:

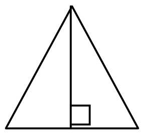


Use S=O/H,C=A/H,T=O/A Sin(angle) = opp/hyp etc.

Opening line	$\sin 27 = \frac{x}{9}$ *x is in the top right position*	$\cos 52 = \frac{7}{x}$ *x is in the bottom right position*	$\tan x = \frac{3}{4}$ *x is the missing angle*

If there is no right angle and we have an isosceles triangle, we can make a right angle by bisecting.





If there is no possibility of using a right angle then we have to use the sine and cosine formulas.

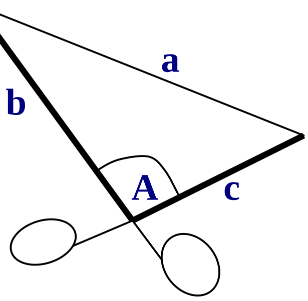
It is advisable to learn these formulas:

The Cosine Formula	$a^2 = b^2 + c^2 - 2bc \cos A$
The Sine formula	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{a}{\sin C}$
The Area of a triangle	½ ab sinC

The Cosine formula

To find a missing side when two sides and the included angle are given

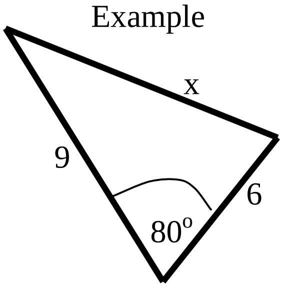
The diagram will look like an open pair of scissors



$$a^2 = b^2 + c^2 - 2bc \cos A$$

The formula starts like Pythagoras but has an **extra bit**

Example



$$x^2 = 9^2 + 6^2 - 2 \times 9 \times 6 \times \cos 80$$

$$x^2 = 81 + 36 - 18.75$$

$$x^2 = 98.25$$

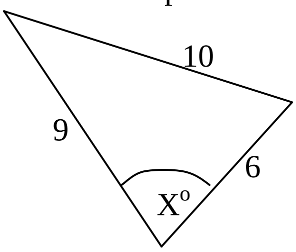
$$x = \sqrt{98.25} = 9.9$$

Note: If the angle is (BIG) greater than 90°, *the cosine of the angle will be negative.*
The extra bit will be positive
$x^2 = 9^2 + 6^2 - 2 \times 9 \times 6 \times \cos 100$ will become $81 + 36 + 18.75$

Note also: If the angle is exactly 90°, *the cosine of 90 will be zero.*
The extra bit will be zero
$x^2 = 9^2 + 6^2 - 2 \times 9 \times 6 \times \cos 0$ will become $81 + 36 + 0$
or **$x^2 = 9^2 + 6^2$** *which brings us back to Pythagoras' Theorem.*

To find a missing angle when three sides and are given

Example



Start in exactly the same way

$$10^2 = 9^2 + 6^2 - 2 \times 9 \times 6 \times \cos X$$

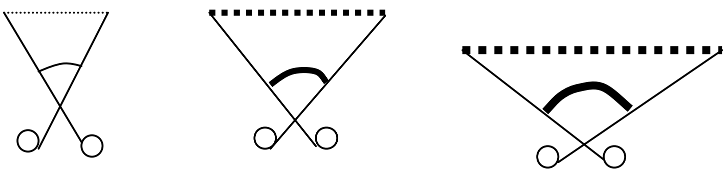
rearrange the formula

$$2 \times 9 \times 6 \times \cos X = 9^2 + 6^2 - 10^2$$

$$108 \times \cos X = 17$$

$$\cos X = \frac{17}{108} \qquad X = \cos^{-1} \frac{17}{108} = 80.9°$$

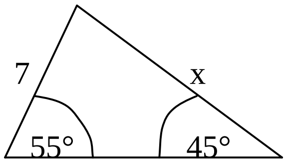
Generally, if the diagram is not a pair of scissors look-alike or if all 3 sides are not given, the triangle will have to be solved using the sine formula.
This formula describes the relationship between a side and its opposite angle.



The bigger the angle the bigger the angle opposite.

The Sine formula

To find a missing side



Special note:

If the angle was 90°
Instead of 45°, **sin 90° = 1**

$$\frac{x}{\sin 55} = \frac{7}{\sin 90}$$

x = 7 X sin 55
which is SOHCAHTOA in action.

There is no right-angle and the diagram does not remind you of a pair of scissors.
One side and two angles are given. (The third angle is 80°).
Use the Sine Formula

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

put the unknown side in the top left position

$$\frac{x}{\sin 55} = \frac{7}{\sin 45}$$

Cross multiply to leave x On its own

$$\frac{x}{1} = \frac{7}{\sin 45} \times \sin 55$$

$$\underline{x = 8.11}$$

To find a missing angle:

The sine formula can be written "upside down"

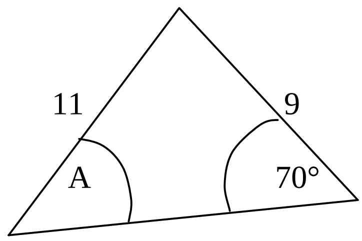
$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

use it this way round and

keep the unknown angle in the **top left position.**
Now cross multiply the 9

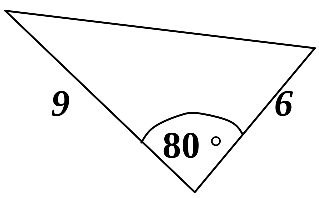
$$\frac{\sin A}{9} = \frac{\sin 70}{11}$$

$$\frac{\sin A}{1} = \frac{\sin 70}{11} \times 9 = 0.7688$$



A = sin -1(0.7688) = <u>50.25°</u>

Back to the scissors:



The <u>area of the triangle</u> *can be found using an alternative to the old* ***half times base times height.***

Use ½ ab sinC ***where a and b are the two sides on either side of the angle.***

Area = ½ x 9 x 6 x sin 80 = 26.6 square units.