HYPOTHESIS TESTING

An explorer claims that the population of rabbits on a remote island are healthy and have a mean weight of 15kg.
An expedition to the island captures 11 adult rabbits and records their weights in kg:
9, 8, 11, 19, 12, 22, 16, 7, 11, 10, 8

We suspect that the mean weight is actually less than 15 and propose to put our theory to the test using statistical methods.

We have a suspicion about the population mean but we will have to estimate the variance using

$$\frac{\sum (x-\bar{x})^2}{n-1} = \frac{\sum (x^2) - n\bar{x}^2}{n-1}.$$

$\bar{x} = \frac{133}{11} = \mathbf{12.1},\quad \hat{\sigma}^2 = \frac{1845-(11\times 12.1^2)}{10} = \mathbf{23.4}$ **So the estimate of the population standard deviation is** $\sqrt{23.4} = \mathbf{4.8} = \hat{\sigma}$

Because the sample size n is small, we use the **t distribution** with (n-1) **10 degrees of freedom**.

$$\frac{\hat{\sigma}}{\sqrt{n}} = \frac{4.8}{\sqrt{11}} = \mathbf{1.45}$$

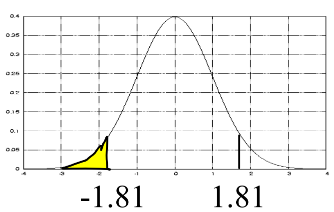
The distribution was first studied by W. S. Gossett, who was a statistician to the Guiness Brewery and wrote undet the pen name “student”.

The distribution is known as “Student,s t distribution”.

Our assumption about the mean is stated as the **null hypothesis**: Ho: $\mu = 15$
The suspician is stated as the alternate hypothesis: H_1: $\mu < 15$

Our test statistic $t = \dfrac{\bar{x}-\mu}{\sqrt{\dfrac{\hat{\sigma}^2}{n}}} = \dfrac{12.1-15}{1.45} = \mathbf{-2}$

This t value has to be well below the mean for us to reject the null hypothesis.
The critical value for this rejection is found from t tables.



The t value found from the sample would have to be less than –1.81 in order that we reject our null hypothesis.

Find the value in t-tables row 10 column 0.95

$t_{95\%}(10) = 1.81$ but

we need to look at the left hand tail.
Since the graph is symmetrical the critical value will be – 1.81.

Since our value of **–2** is less than the critical value **-1.81** **we reject** Ho at the 5% significance level and conclude that the mean is less than 15.

A 95% confidence interval for μ is now $\bar{x} \pm t_{95\%}(10)\dfrac{\hat{\sigma}}{\sqrt{n}}$

= **12.1** ± 1.81x**1.45** = (9.5, 14.7).

Confidence intervals for a proportion.

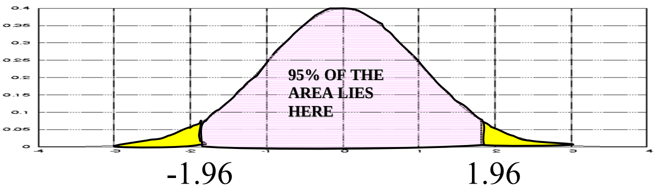
In the following example it is not the mean but the population proportion that is unknown.

In simple probability questions we are told that the probability of some event is $\frac{a}{b}$**, but how do they know that?**

How do they know that the probability that the fish in the lake has some fishy-type disease? Suppose they take a sample of 500 fish and find that120 have the particular disease. It would be easy to say that "The probability of a fish having the disease is $\frac{120}{500}$. And no disease $\frac{380}{500}$.

But how confident are we of the statement considering that we made it on the strength of a sample which would be different on another day?

It is better to give a range for that probability with a stated degree of confidence. **So we give a confidence interval for the proportion.**



95% confidence interval (0.203, 0.277)

99% confidence interval (0.191, 0.289) a little wider.

The confidence interval is quoted directly from the sample proportion and using the normal distribution. It is similar to a confidence interval for a normal distribution.

With the sample proportion p = $\frac{120}{500}$ = **0.24**

and q = $\frac{380}{500}$ = **0.76**

A 95% confidence interval for the population proportion is $\frac{120}{500} \pm \mathbf{1.96}\sqrt{\frac{0.24x0.76}{500}}$

For a 99% confidence interval replace the 1.96 by 2.58.