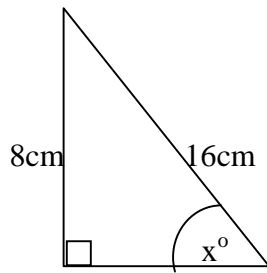


SOLVING TRIG EQUATIONS

A simple trigonometry equation would have arisen from this triangle:



$$\sin x = \frac{8}{16} = \frac{1}{2} \quad x = \sin^{-1} \frac{1}{2} = 30$$

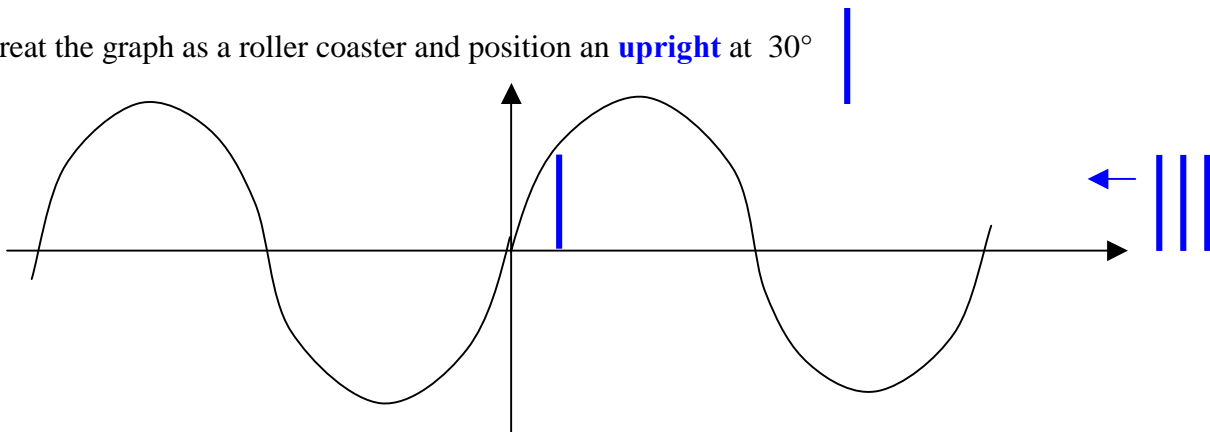
The solution of $\sin x = \frac{1}{2}$ is $x = 30^\circ$

To solve $\sin x = \frac{1}{2}$ where all solutions in the range $-360^\circ \leq x \leq 360^\circ$ are required:

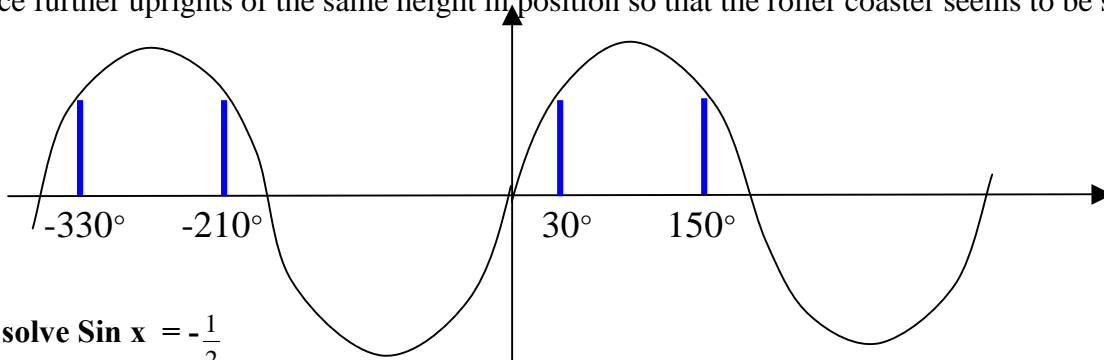
Solve the equation as above to get $x = 30^\circ$

Sketch a good-sized sine graph.

Treat the graph as a roller coaster and position an **upright** at 30°

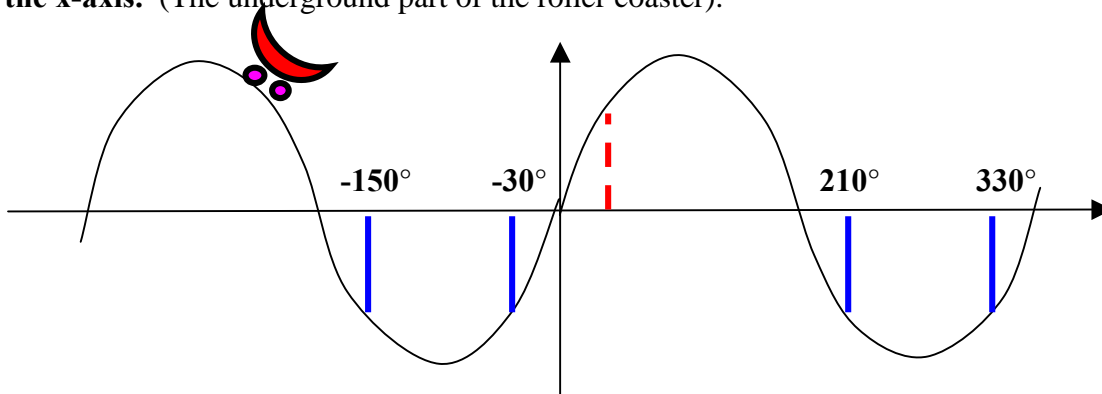


Place further uprights of the same height in position so that the roller coaster seems to be supported.



To solve $\sin x = -\frac{1}{2}$

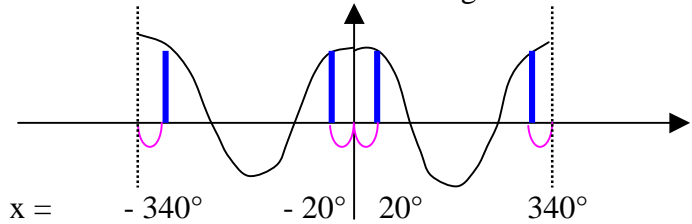
Ignore the negative sign, proceed as if you are solving $\sin x = \frac{1}{2}$ but position the uprights **below the x-axis**. (The underground part of the roller coaster).



To solve $\cos x = 0.94$ for $-360^\circ \leq x \leq 360^\circ$

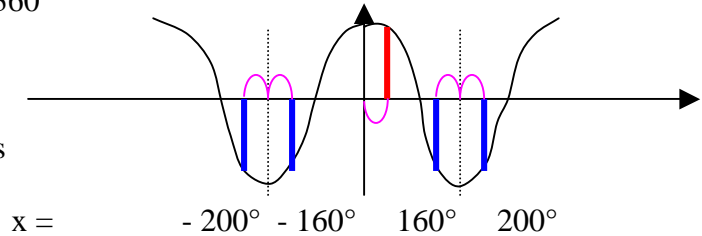
- Sketch the graph if $y = \cos x$
- Position the first upright at 20°
- Position all other uprights
- Mark the angles where they occur.
- These are the solutions to the equation:

Find $\cos^{-1}(0.94)$
This is 20° to the nearest degree.



To solve $\cos x = -0.94$ for $-360^\circ \leq x \leq 360^\circ$

- Sketch the graph if $y = \cos x$
- Position the first upright at 20°
- Position all other uprights below the x -axis
- Mark the angles where they occur.
- These are the solutions to the equation:



If the range for the solutions is different to $-360^\circ \leq x \leq 360^\circ$ and is perhaps $-180^\circ \leq x \leq 180^\circ$ Sketch the whole graph anyway but only quote answers in the required range. $x = -160^\circ, 160^\circ$

To solve $\tan(2x) = 1.732$ We could always start by sketching the graph of $\tan(2x)$

It is probably easier to sketch the graph of $y = \tan x$.
Let the $2x = t$ and solve $\tan(t) = 1.732$ first.

$$\tan^{-1}(1.732) = 60^\circ$$

Put the other values in.

$$t = -300^\circ, -120^\circ, 60^\circ, 240^\circ$$

but $t = 2x$

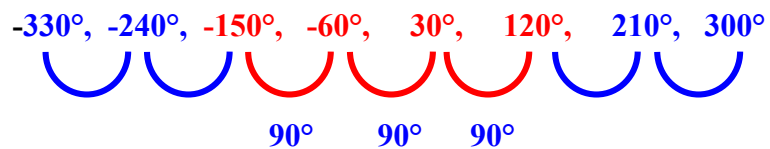
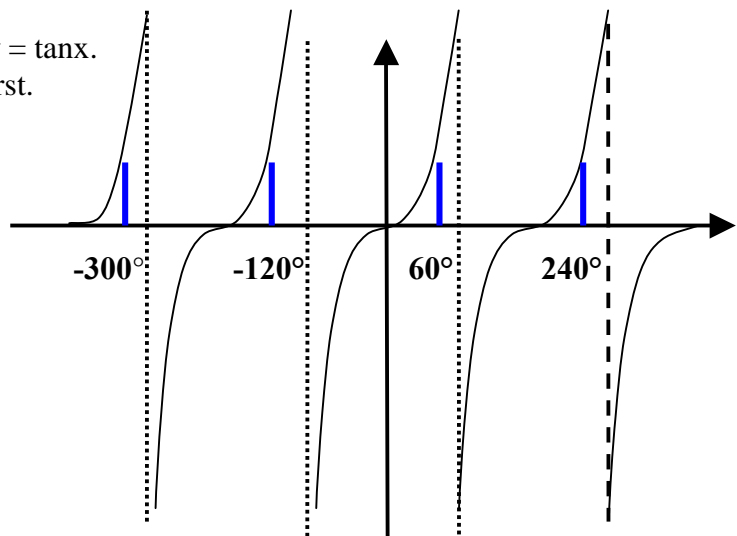
$$2x = -300^\circ, -120^\circ, 60^\circ, 240^\circ$$

so

$$x = -150^\circ, -60^\circ, 30^\circ, 120^\circ$$

These values now fall within half the range; $-180^\circ \leq x \leq 180^\circ$

To give all answers within the original range simply continue the sequence of answers.



To solve $3\cos x = 1$:

Write the equation as $\cos x = \frac{1}{3}$ and proceed as above.

To find maximum and minimum values sketch the graphs and read from your sketch.