VαRIATION

It has been known for some time that the circumference of a circle is equal to π times the diameter.

C = π d.

If we didn't know this we may suspect that the circumference is related to the diameter and say that the circumference is proportional to the diameter. The circumference is "something" times the diameter.
Knowing that the **circumference is proportional to the diameter**, we write: C α d
We can't work with this symbol so we change to an equation: C = kd
k is known as the **constant of proportionality.**
We now need some evidence from an experiment…….Suppose we measured the diameter of a plate to be 20cm and (by placing a length of string round the rim of the plate) the circumference is 60cm.

This is the information we need: when d = 20, C = 60.
Substitute into the equation: 60 = k 20
Solving this for k gives k = 3.
We can now write the equation and use it to solve problems for all circles. C = 3d

Tackling questions on variation or proportionality:

The opening statement:	**A** is proportional to **b**	**P** varies as **q**	**C** varies in direct proportion to **d**	**F** varies directly as **g**
Write the statement using the proportionality symbol α:	A α b	P α q	C α d	F α g
Change to an equation using the constant of proportionality k:	A = kb	P = kq	C = kd	F = kg
Read on for clues:	When b = 3, A = 12	Given that P = 10 when q = 4	d: 5, – C: 15, 21	When g = 4, F = 9
Substitute and find the value of k:	12 = k3 k = 4 **A = 4b**	10 = k4 k = 2½ **P = 2½ q**	15 = k5 k = 3 **C = 3d**	9 = k4 $k = \frac{9}{4}$ $\mathbf{F} = \frac{9}{4}\mathbf{g}$
We can now find any required values using the equation:	Find the value of A when b = 10	Calculate P when q = 6	Find **d** when **C** = 21	Calculate F when g = 16
Solving:	**A = 4 x 10** **= 40**	**P = 2½ x 6** **=15**	**21 = 3d** **d = 7**	$\mathbf{F} = \frac{9}{4}\mathbf{x16}$ **= 36**

The above are all examples of two quantities being in direct proportion. The equations will all be straight lines.
Quantities may also be in **inverse proportion** to one another. When one increases, the other decreases.

If A is inversely proportional to b we write A α $\frac{1}{b}$ and continue with the equation $\mathbf{A = k}\ \frac{1}{b}$

This could also be written $\mathbf{A} = \frac{k}{b}$ **where k can be found when we substitute for A and b.**

If it takes two men 6 hours to dig a hole. How long will it take four men to dig the same size hole?

If we took the time as being proportional to the number of men on the job: $t \propto n$ and continued:
$t = kn$, $6 = k\,2$, $k = 3$, **$t = 3n$**, when $n = 4$ $t = 12$ 12 hours?

This is clearly not correct. Since there are more men on the job, it should take less time. This is an example of **inverse proportion** and gives the correct answer if we proceed as follows: $t \propto \frac{1}{n}$

$t = \frac{k}{n}$, $6 = \frac{k}{2}$, $k = 12$, **$t = \frac{12}{n}$**, when $n = 4$ $t = 3$ **i.e. 3 hours!**

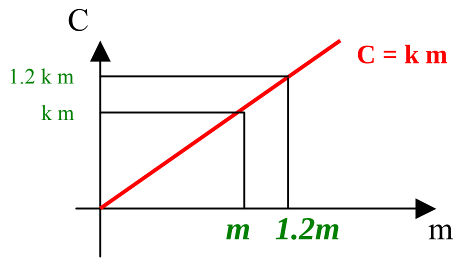
(I know that four men have more to chat about and have more cups of tea).

As well as two quantities being in direct or inverse proportion to one another, one quantity may be proportional to the square, the cube or the square root of the other.
Read the opening statement carefully!

The opening statement:	**A** is proportional to the square of **b**	**P** varies inversely as **q**	**C** varies inversely as the cube of **d**	**F** is directly proportional to the square root of **g**
Write the statement using the proportionality symbol α:	$A \propto b^2$	$P \propto \frac{1}{q}$	$C \propto \frac{1}{d^3}$	$F \propto \sqrt{g}$
Change to an equation using the constant of proportionality k:	$A = k\,b^2$	$P = k\,\frac{1}{q}$	$C = k\frac{1}{d^3}$	$F = k\,\sqrt{g}$
Read on for clues:	When b = 3, A = 18	Given that P = 10 when q = 4	d: 5; C: 1	When g = 4, F = 9
Substitute and find the value of k:	$\mathbf{A = 2b^2}$	$\mathbf{P = 40\,\frac{1}{q}}$	$\mathbf{C = 125\frac{1}{d^3}}$	$\mathbf{F = 4.5\,\sqrt{g}}$

What if there are no clues?

The cost of a box of cereal is directly proportional to the mass of the cereal in the box. When the mass is increased by 20% find the percentage increase in the cost.

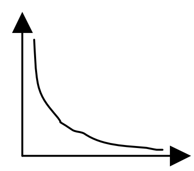


Direct proportion is represented by a straight line graph.
Take an initial value ***m*** for the mass. The value of C will be **k*m***.
A 20 % increase in m gives the point **1.2 *m***. and **C = k 1.2 *m***

The change in C is **k 1.2 *m* - k *m* = 0.2 k *m***

The percentage change is $\frac{0.2km}{km} \times 100$ = **20%.**

The cost is increased by the same percentage as the mass in this question and all questions on direct proportion.



The same problem for inverse proportion gives a **16.7% decrease**.

Use the sketch $\mathbf{C} = \frac{k}{m}$ to check this.