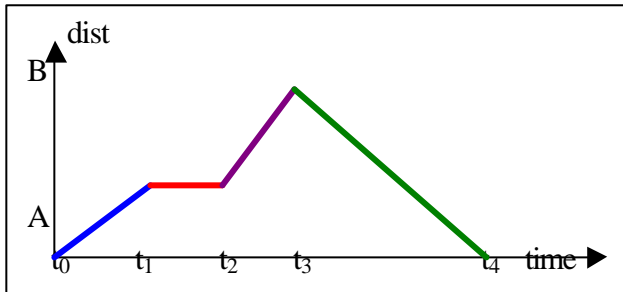


VELOCITY – TIME - DISTANCE



Particles moving in a straight line could be the model for a marble rolling down a groove, a train travelling between stations or an object being thrown from a building and accelerating under the action of gravity. The motion can be represented graphically and the problem can be solved using graphical methods, standard equations of motion or calculus methods.

A simple distance/time graph would look like:



The graph shows the journey from A to B, along a straight road and back to A again. The **distance** corresponding to any time value can be seen from the graph and at time t_4 , the distance is zero so the particle is back at A.

The **gradient**, being the value of $\frac{dist}{time}$ is the velocity and it is;

Zero between t_1 and t_2 . No speed or a rest stop.

Steeper between **t_2 and t_3** than between **t_0 and t_1** showing greater velocity between **t_2 and t_3** .

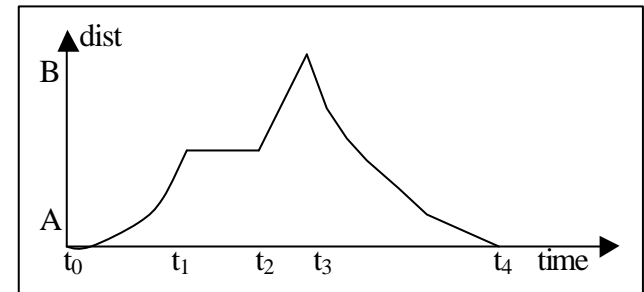
Between **t_3 and t_4 the particle is returning and this is shown as a negative gradient.**

The straight lines indicate **constant** velocity.

Curves on the graph indicate variable velocity or acceleration and have to be given as an equation.

For example: The distance s for the first part of the journey is given by the formula $s = t^2$
The gradient is increasing and we can find the gradient of the curve at a point.

This is where calculus comes in $\frac{ds}{dt} = 2t$ i.e. $v = 2t$, v being the velocity for any value of t .



Differentiate distance to get velocity, velocity to get acceleration.

$S \longrightarrow V \longrightarrow a$

Integrate acceleration to get velocity, velocity to get distance.

$a \longrightarrow V \longrightarrow S$

Put in the limits when integrating
 $V = 6 + 3t^2$. Find distance travelled
between $t = 1$ and $t = 3$

$$S = \int_1^3 6 + 3t^2 dt = [6t + t^3]_1^3 = 38$$

QUESTIONS.

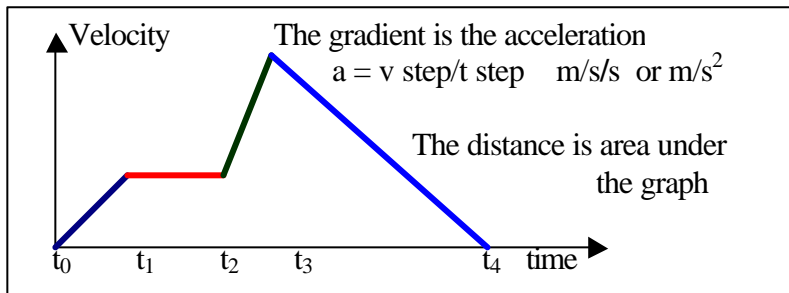
TYPE 1 will be given in terms of an equation and the solution is to be found using calculus:

If $s = f(t)$ then $\frac{ds}{dt}$ will give the equation for the velocity $v = f'(t)$. From here $\frac{dv}{dt}$ will give the equation for acceleration $a = f''(t)$

If the question gives the equation for acceleration: integrate to get velocity $\int_{t_1}^{t_2} (a)dt = v$. From here $\int_{t_1}^{t_2} (v)dt$ gives distance s .

TYPE 2 will describe the journey and the solutions may be found by graphical methods or using the equations of motion.

Use a velocity/time graph



OR: Use the equations of motion for constant acceleration:

If the initial velocity is u
 If the final velocity is v
 If the distance is s
 If the acceleration is a

Use one of:

$$v = u + at$$

$$v^2 = u^2 + 2as$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{(u + v)t}{2}$$

Example:

<p>The train accelerates from rest to a speed of 40 m/s From A to B over a distance of 1 km. (1000m) The distance is the <u>area</u> of a triangle: $\frac{1}{2} \times t \times 40 = 1000$ $t = 50$ secs acceleration = $40/50 = 0.8$</p> <p>OR: $u = 0, v = 40, t = ?, a = ?$ $v^2 = u^2 + 2as$ $a = 0.8$ $v = u + at$ $t = 50$</p>	<p>It then travels a distance of 2 km at this speed. The distance is the area of a rectangle. $40t = 2000, t = 50$ Horizontal line accel = 0</p> <p>OR: $u = 40, v = 40, s = 2000$ $40^2 = 40^2 + 2a(2000), a = 0$ $s = ut + \frac{1}{2}at^2, 2000 = 40t, t = 50$</p>	<p>Then it accelerates at 4 m/s^2 until it reaches a speed of 100 m/s The distance is the area of a trapezium. The gradient is $a = (100 - 40)/t$ $4 = 60/t, t = 15$</p> <p>$S = \frac{1}{2} (40 + 100)15 = 1050$ OR: $a = 4, u = 40, v = 100$ $V = u + at$ $100 = 40 + 4t, t = 15$ $s = ut + \frac{1}{2}at^2$ $s = 40 \times 15 + 2 \times 15^2$ $s = 1050$</p>	<p>Finally it decelerates uniformly For 4km until it stops.</p> <p>The distance is the <u>area</u> of a triangle: $\frac{1}{2} \times t \times 100 = 4000$ $t = 80$ secs deceleration = $100/80 = 1.25$</p> <p>OR: $u = 100, v = 0, t = ?, a = ?$ $v^2 = u^2 + 2as$ $a = -1.25$ deceleration. $v = u + at$ $t = 80$</p>
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The total time is $50 + 50 + 15 + 80 = 195$ secs. The total distance is the sum of the 4 distances.